A Model for Finding Recoverable Robust Periodic Timetables

V. Grafe[a,*], A. Schöbel[a,b]

[a] Department of Mathematics, Technische Universität Kaiserslautern, Kaiserslautern, Germany
grafe@mathematik.uni-kl.de, schoebel@mathematik.uni-kl.de

[b] Fraunhofer-Institute for Industrial Mathematics ITWM, Kaiserslautern, Germany

[*] Corresponding author

Extended abstract submitted for presentation at the 11th Triennial Symposium on Transportation Analysis conference (TRISTAN XI)
June 19-25, 2022, Mauritius Island

March 30, 2022

Keywords: Periodic Timetabling, Delay Management, Recoverable Robustness, Mixed Integer Programming

1 MOTIVATION

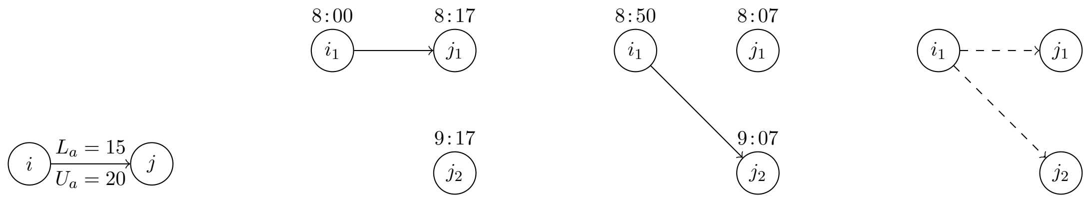
An important aspect of optimising public transport is finding a good timetable. Often a *periodic* timetable is desirable, i.e. a timetable which repeats in a regular pattern (e.g. every hour). On the one hand, short travelling times are important from the passengers' point of view. The problem of finding a periodic timetable with minimal travelling times is known as the *Periodic Event Scheduling Problem (PESP)* and is well researched, see e.g. Liebchen (2006). On the other hand, tight timetables without buffer times are prone to delays, which are inevitable in practice and highly dissatisfactory for the passengers. Hence, a good timetable should also have some degree of delay resistance. Many concepts and ideas on how to increase the robustness of a timetable against delays exist, see e.g. Lusby *et al.* (2018). However, none of these approaches uses the promising concept of *recoverable robustness* introduced by Liebchen *et al.* (2009), although the literature on delay management is dedicated to recovering timetables.

In our work we combine timetabling and delay management to be able to find recoverable robust timetables. We develop a mixed integer programming formulation for the *Recoverable Robust Periodic Timetabling Problem (RRPT)*. Furthermore, we identify similarities and differences to strictly robust timetables and analyse the relation between the nominal travelling time and the delays. We also tackle the challenge of finding heuristic solutions.

2 PROBLEM DESCRIPTION

We are given a scenario set $\mathcal{U}$, where every scenario $r \in \mathcal{U}$ is a set of source delays. A timetable π is called (α, β)-recoverable-robust w.r.t. $\mathcal{U}$ if for every $r \in \mathcal{U}$ there is a disposition timetable such that the total delay and the number of missed transfers (both weighted with the number of passengers) are bounded by α respectively β. The *Recoverable Robust Periodic Timetabling Problem* is the problem of finding a periodic timetable which is (α, β)-recoverable-robust and minimises the nominal travelling time summed over all passengers.

We briefly describe the PESP, which is a common model for periodic timetabling. In the PESP we are given a period T together with a set of events $\underline{\mathcal{E}}$. Each event corresponds to the arrival or the departure of a traffic line at some station. Furthermore, we have activities $\underline{\mathcal{A}} = \underline{\mathcal{A}}_{\text{train}} \cup \underline{\mathcal{A}}_{\text{transfer}}$



(a) *Periodic activity* (b) *Rolled out activity for one timetable* (c) *Rolled out activity for another timetable* (d) *Both possible rolled out activities*

Figure 1 – *Problem of rolling out a periodic EAN without knowing the timetable*

which represent driving and waiting of trains (between and in stations) and transfers of passengers. We obtain an *event-activity-network* (EAN) $\underline{\mathcal{N}} = (\underline{\mathcal{E}}, \underline{\mathcal{A}})$. Every activity has a lower bound L_a and an upper bound U_a for its duration. Additionally, w_a is the number of passengers using activity $a \in \underline{\mathcal{A}}$ and w_i is the number of passengers arriving at their destination $i \in \underline{\mathcal{E}}$. A timetable with period T assigns a time $\pi_i \in \{0, \dots, T-1\}$ to every event. Events are repeated every T minutes. The objective is to minimise the total travelling time.

In practice, delays may occur and make a timetable infeasible. A set of source delays is called a *scenario*. The *Delay Management problem (DM)* consists of two tasks: finding a disposition timetable, i.e. a new timetable which respects the source delays, and deciding which transfers should be maintained and which can be cancelled. See Dollevoet *et al.* (2018) for a survey. As delays do usually not occur periodically, DM is not considered in the periodic EAN, but in a aperiodic (non-periodic) network. Here, an event does not represent the arrival or departure of a *line* (which repeats every T minutes), but of a single *trip*. DM usually has a timetable as input. This timetable is used to *roll out* the periodic EAN $\underline{\mathcal{N}} = (\underline{\mathcal{E}}, \underline{\mathcal{A}})$ to the aperiodic network $\mathcal{N} = (\mathcal{E}, \mathcal{A})$ for some given planning horizon $I = [t_{\min}, t_{\max}]$. If K is the number of periods in I, every periodic event $i \in \underline{\mathcal{E}}$ has K corresponding aperiodic events $i_1, \dots, i_K$ in the planning horizon to which we add $b := \lceil \max_{a \in \underline{\mathcal{A}}} U_a / T \rceil$ further events to make sure that also the arcs leaving I can be considered. The set of activities $\mathcal{A}$ depends on the timetable, as can be seen in the small example in Figure 1.

The goal of our paper is to integrate PESP and DM. PESP is a periodic problem in the EAN, while Delay Management uses the aperiodic network $\mathcal{N}$. For the integrated problem we have to use the same network. We decided to use the aperiodic network $\mathcal{N}$. Finding a periodic timetable in an appropriately rolled out aperiodic network is called *Periodic Timetabling in Aperiodic Network (PTTA)* and was introduced in Grafe & Schöbel (2021), where it was also shown to be equivalent to PESP. It requires solving an assignment problem to choose activities for the timetable (in Figure 1(d) one of the two dashed activities has to be chosen).

3 MODELLING

We develop a mixed integer programming formulation. The objective function minimises the nominal travelling time summed over all passengers. The constraints can be divided into two subproblems as indicated by the boxes. For the master problem PTTA of finding a periodic timetable in the aperiodic network $\mathcal{N}$ (see Grafe & Schöbel (2021)) we need timetabling variables π_i for $i \in \mathcal{E}$ and assignment variables u_a for $a \in \mathcal{A}$. (1) and (2) ensure that the chosen activities respect the upper and lower bounds. The synchronisation constraints (3) enforce periodicity. The assignment problem is included by (4). The auxiliary variables F_a determinie the travel time needed in the objective function, see (5). Note that due to the periodicity of the timetable it suffices to have these variables only for one period, i.e. we only define them for arcs of the form $a = (i_1, j_t)$. (6) and (7) ensure that the timetable is indeed within the planning horizon. We have to solve the delay management problem in every scenario. As in Schöbel (2007),

$$\min \sum_{a=(i_1,j_t)\in\mathcal{A}} w_a F_a \cdot K \qquad \text{(RRPT}(\mathcal{U},\alpha,\beta))$$

PTTA

$$
\begin{array}{lll}
\pi_j - \pi_i + M(u_a - 1) \leq U_a & a = (i,j) \in \mathcal{A} & (1) \\
\pi_j - \pi_i + M(1 - u_a) \geq L_a & a = (i,j) \in \mathcal{A} & (2) \\
\pi_{i_s} - \pi_{i_{s-1}} = T & i_s \in \mathcal{E}, 2 \leq s \leq K + b & (3) \\
\sum_{t:a'=(i_s,j_t)\in\mathcal{A}} u_{a'} = 1 & a = (i,j) \in \underline{\mathcal{A}}, 1 \leq s \leq \ K & (4) \\
F_a \geq M(u_a - 1) + \pi_{j_t} - \pi_{i_1} & a = (i_1, j_t) \in \mathcal{A} & (5) \\
\pi_i \geq t_{\min} & i \in \mathcal{E} & (6) \\
\pi_{i_1} \leq t_{\min} + T - 1 & i \in \underline{\mathcal{E}} & (7) \\
\pi_i \in \mathbb{N} & i \in \mathcal{E} & (8) \\
u_a \in \{0,1\} & a \in \mathcal{A} & (9) \\
F_a \geq 0 & a = (i_1, j_t) \in \mathcal{A} & (10)
\end{array}
$$

DM

$$
\begin{array}{lll}
x_i^r \geq \pi_i + d_i^r & i \in \mathcal{E}, r \in \mathcal{U} & (11) \\
M'(1 - u_a) + x_j^r - x_i^r \geq L_a + d_a^r & a = (i,j) \in \mathcal{A}_{\text{train}}, r \in \mathcal{U} & (12) \\
M'(1 - u_a) + M' y_a^r + x_j^r - x_i^r \geq L_a & a = (i,j) \in \mathcal{A}_{\text{transfer}}, r \in \mathcal{U} & (13) \\
\sum_{a\in\mathcal{A}_{\text{transfer}}} w_a y_a^r \leq \beta & r \in \mathcal{U} & (14) \\
\sum_{i_s\in\mathcal{E}:s\leq K} w_{i_s}(x_{i_s}^r - \pi_{i_s}) + \sum_{a\in\mathcal{A}_{\text{out}}} w_a H_a^r \leq \alpha & r \in \mathcal{U} & (15) \\
H_a^r \geq M''(u_a - 1) + x_j^r - \pi_j & a \in \mathcal{A}_{\text{out}}, r \in \mathcal{U} & (16) \\
x_i^r \in \mathbb{N} & i \in \mathcal{E}, r \in \mathcal{U} & (17) \\
y_a^r \in \{0,1\} & a \in \mathcal{A}_{\text{transfer}}, r \in \mathcal{U} & (18) \\
H_a^r \geq 0 & a \in \mathcal{A}_{\text{out}}, r \in \mathcal{U} & (19)
\end{array}
$$

in the DM subproblem, for every $r \in \mathcal{U}$ we need variables x_i^r determining the time of event $i \in \mathcal{E}$ in the disposition timetable and binary variables y_a^r for every transfer activity a, determining if the transfer is maintained. An event can not take place earlier than in the original timetable π (plus some possible source delay), as enforced by (11). For the driving and waiting activities $\mathcal{A}_{\text{train}}$ as well as for the maintained transfer activities $\mathcal{A}_{\text{transfer}}$ the disposition timetables have to fulfil the lower bound (plus source delay), which is ensured by (12) and (13). The total delay over all passengers and the number of missed transfers are bounded by α respectively β in (15) and (14). (16) give special attention to arcs leaving the planning horizon. Note that nearly all constraints of (DM) are coupling constraints and that we use different sizes of big-M-constraints.

4 ANALYSIS AND OUTLOOK

For the concept of strict robustness it is known that a robust feasible solution remains robust feasible if the uncertainty set is extended to its convex hull (Ben-Tal *et al.*, 2009). Unfortunately, for recoverable robustness this is not true in general as demonstrated in Carrizosa *et al.* (2017). However, we can show this property for a special case:

Lemma 1. *If π is $(\alpha, 0)$-recoverable-robust w.r.t. $\mathcal{U}$, then it is also $(\alpha, 0)$-recoverable-robust w.r.t.* $\operatorname{conv}(\mathcal{U})$*, if the integrality constraint on x is relaxed.*

Proof. Since π is $(\alpha, 0)$-recoverable-robust w.r.t. $\mathcal{U}$, for every $r \in \mathcal{U}$ there is a DM-solution $(x^r, 0)$ with delay bounded by α. In particular, we have $x_j^r - x_i^r \geq L_a + d_a^r$ for all $a = (i, j) \in \mathcal{A}_{\text{train}} \cup \mathcal{A}_{\text{transfer}}$. Let $\bar{r} \in \operatorname{conv}(\mathcal{U})$, i.e. all source delays are of the form $d_a^{\bar{r}} = \sum_{r \in \mathcal{U}} \lambda_r d_a^r$ for $a \in \mathcal{A}_{\text{train}}$ resp. $d_i^{\bar{r}} = \sum_{r \in \mathcal{U}} \lambda_r d_i^r$ for $i \in \mathcal{E}$. For every $i \in \mathcal{E}$ define $x_i^{\bar{r}} := \sum_{r \in \mathcal{U}} \lambda_r x_i^r$ and set $y_a^{\bar{r}} = 0$ for all $a \in \mathcal{A}_{\text{transfer}}$. It follows for $a = (i, j) \in \mathcal{A}$:

$$x_j^{\bar{r}} - x_i^{\bar{r}} = \sum_{r \in \mathcal{U}} \lambda_r (x_j^r - x_i^r) \geq \sum_{r \in \mathcal{U}} \lambda_r (L_a + d_a^r) = \sum_{r \in \mathcal{U}} \lambda_r L_a + \sum_{r \in \mathcal{U}} \lambda_r d_a^r = L_a + d_a^{\bar{r}} \tag{20}$$

Analogously, we obtain $x_i^{\bar{r}} \geq \pi_i + d_i^{\bar{r}}$ for $i \in \mathcal{E}$. The total delay is

$$\begin{aligned} & \sum_{i \in \mathcal{E}} w_i (x_i^{\bar{r}} - \pi_i) + \sum_{a=(i,j) \in \mathcal{A}_{\text{out}}} w_a (x_j^{\bar{r}} - \pi_j) \\ = & \sum_{r \in \mathcal{U}} \lambda_r \underbrace{\Big(\sum_{i \in \mathcal{E}} w_i (x_i^r - \pi_i) + \sum_{a=(i,j) \in \mathcal{A}_{\text{out}}} w_a (x_j^r - \pi_j) \Big)}_{=Z^r} \leq \max_{r \in \mathcal{U}} Z^r \leq \alpha \end{aligned} \tag{21}$$

□

In particular, we show that for integral input data and $\beta = 0$ it suffices to solve RRPT only for the extreme points of the scenario set, i.e. if we require that all transfers are maintained, we can reduce our problem size considerably. We further analyse the relation between α, β and the objective function value: Requiring only few delay leads to bad nominal travelling times.

The integer program is too large to be solved by a solver even for small instances. However, timetabling and DM can be tackled, which is the basis of iterative and decomposition approaches. We develop an iterative heuristic which is tested on close-to real world examples from the LinTim library (Schiewe *et al.*, 2021).

References

Ben-Tal, Aharon, El Ghaoui, Laurent, & Nemirovski, Arkadi. 2009. *Robust Optimization.* Princeton University Press.

Carrizosa, Emilio, Goerigk, Marc, & Schöbel, Anita. 2017. A biobjective approach to recovery robustness based on location planning. *European Journal of Operational Research*, **261**, 421–435.

Dollevoet, Twan, Huisman, Dennis, Schmidt, Marie, & Schöbel, Anita. 2018. Delay propagation and delay management in transportation networks. *Pages 285–317 of: Handbook of Optimization in the Railway Industry.* Springer.

Grafe, Vera, & Schöbel, Anita. 2021. Solving the Periodic Scheduling Problem: An Assignment Approach in Non-Periodic Networks. *Pages 9:1–9:16 of: ATMOS 2021.* Open Access Series in Informatics (OASIcs), vol. 96. Dagstuhl – Leibniz-Zentrum für Informatik.

Liebchen, Christian. 2006. *Periodic timetable optimization in public transport.* Ph.D. thesis, TU Berlin.

Liebchen, Christian, Lübbecke, Marco E., Möhring, Rolf H., & Stiller, Sebastian. 2009. The concept of recoverable robustness, linear programming recovery, and railway applications. *Lecture Notes in Computer Science*, **5868**, 1–27.

Lusby, Richard M., Larsen, Jesper, & Bull, Simon. 2018. A survey on robustness in railway planning. *European Journal of Operational Research*, **266**, 1–15.

Schiewe, Alexander, Albert, Sebastian, Grafe, Vera, Schiewe, Philine, Schöbel, Anita, & Spühler, Felix. 2021. *LinTim: An integrated environment for mathematical public transport optimization. Documentation for version 2021.12.* Tech. rept. Fraunhofer-Institut für Techno- und Wirtschaftsmathematik.

Schöbel, Anita. 2007. Integer Programming approaches for solving the delay management problem. *Pages 145–170 of: Algorithmic Methods for Railway Optimization.* Lecture Notes in Computer Science, no. 4359. Springer.