Sequential transit route design by link expansion using knowledge gradient with correlated beliefs

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1 INTRODUCTION

Mobility service route design depends upon identifying the demand level in the potential service region of the system. In conventional transportation planning, an operator would collect survey data from travelers in the region and use that to forecast the demand. However, one of the shortcomings of surveys is the limited sampling rates. For example, the 2010/2011 Regional Household Travel Survey (RHTS) of the New York Metropolitan Transportation Council sampled 18,965 households, smaller than the entire population of about 7 million households (Census Reporter, 2021) in the area. In addition, the dataset may not be useful when it comes to emerging transit technologies that have never operated in the region before, such as modular autonomous vehicle fleets (e.g. Caros and Chow, 2020) or more elaborate feeder-trunk route systems that make use of various emerging shared mobility options (e.g. Ma et al., 2019).

Operated routes of a mobility service can act as sensors for valuable data generated during the operation, including vehicle trajectory, load profile, revealed preferences, ridership, average wait time, and user ratings. Routes should be implemented with explicit consideration of impacts on the service provided to travelers and the knowledge gained. In this way, an operator can use these data to design their route system to fit the prevailing demand in a sequential or phased implementation (Yoon and Chow, 2020) instead of implementing a single design at once. This explicit consideration of route design as sensors is called transit service route design with “optimal learning.”

In optimal learning, sequential decisions are made such that the information gained each decision epoch is optimized to reduce the uncertainty for subsequent decisions. This study proposes a new approach to design mobility service routes tailored to local demand patterns by adapting a learning-based scheme. Due to the dependence between choices in a network setting, the knowledge gradient (Frazier et al., 2008) could be a suitable method because it does not require an assumption of independence between options. However, this can still be problematic because of the way the covariance matrix between different route elements scale up (Ryzhov et al., 2012).

We propose several contributions to resolve this issue for transit route design. First, the proposed framework clarifies how the procedure of sequential route design can be formulated. It also describes the flow of data to illustrate the inputs required and outputs being developed. Second, different learning policies are compared based on two performance measures: total covered demand, and optimal choice rate. We study three different learning policies to identify the conditions where one policy is most appropriate.

2 METHODOLOGY

2.1 Problem Definition

First, let us clarify the concepts of "extension" and "expansion" in a transit network. An extension appends a link to a current route and elongates the route length. After a few observations, new information is obtained from users served by that additional link. A route can be built by repeating until the predefined maximum length. On the contrary, an expansion adds a new route to an existing route set and improves the coverage of the system. This approach can benefit operators in several ways: improvement of current knowledge, prompt responses to dynamic demand, and dealing with sequential budget investments.

Consider a system that is gradually expanded on a network $G(N, A)$. Flows of $N(N-1)$ available OD pairs on G are denoted as $x_{o,d}$, where o is the origin and d the destination. When a k-th route R_k consists of l nodes, it can serve $l(l-1)$ OD flows alone. This coverage grows if there is another route intersecting with R_k, allowing potential passengers to travel further by transfer. For each decision epoch t, a link a^t is chosen from a set A^t to append to $\{R_k\}_{k\in K}$, where the set of corresponding nodes c^t is defined as $C^t \subset N$. The algorithm evaluates a^t by v_{a^t}, the value of a^t that consists of stochastic reward and exploration term that represents the potential benefit of exploration by collecting additional information from the option. A deterministic version of the network expansion problem is shown in Fragkos et al. (2021).

$$a^{t\prime} = \arg\max_{a^t \in A^t} v_{a^t} \quad (1)$$

Various policies use different structures of values for the evaluation, and this study compares Knowledge Gradient (KG), KG with Correlated Belief (KGCB), and Multi-Armed Bandit (MAB). While these learning algorithms already exist, a network design algorithmic framework is needed to make use of them properly. In general, $a^{t\prime}$, the optimum among available a^t can be found by Eq. (1). After the evaluation, R_k is extended to $a^{t\prime}$, and the next extension at $t+1$ is conducted. This repeats until the termination criteria are satisfied, and the generation of the next route begins while the budget remains. For example, if L is the maximum number of nodes covered by R_k, it will have a sequence of nodes $\{n_{R_k,1}, n_{R_k,2}, \cdots, n_{R_k,L}\}$. The next route R_{k+1} will begin from $n_{R_{k+1},1}$.

2.2 Sequential transit route design algorithmic framework

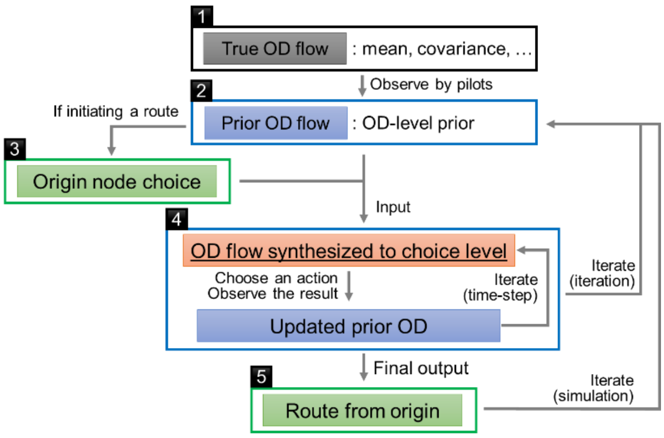


Figure 1 – Framework of learning-based link-level extension route system design

Figure 1 describes the algorithmic framework. Box 1 represents unknown true reward. Therefore, the sequential route design is initiated with a sampling, i.e. the pilot project(s), to build a base knowledge (or prior) about them, described in Box 2. Box 3 show a designation of the first node to initiate link-based route extension. During the extension, only links that can be appended to a currently extending route are included in the set. In Box 4, the illustration shows the aggregation of individual choices to create link-level knowledge. It returns to Box 2 until the current route reaches the maximum distance, stopping the extension. Box 5 is the final output of a route extension, and the procedure returns to Box 2 if there are more routes to be designed. Note the difference between the inner and outer loops. The inner loop in Box 4 is represents simulated scenarios used to feed the learning algorithms to determine the link expansion in one epoch. The outer loop between Box 2 and 4 is the sequential transit route extensions conducting over multiple epochs taking place in real time.

2.3 Numerical Experiment

Two sets of numerical experiments are conducted to test the proposed sequential route design framework: an artificial 10-by-10 grid network and a real-world example based on New York City (NYC) data.

First, the generated grid includes 100 nodes and 360 unidirectional links. The prior mean vector has 9,900 elements, representing the number of observable OD flows on this network (100×99). True mean OD flows are randomly generated based on a gravity model, defining the amount of OD flow between two zones as the product of zonal attributes and the inverse of the power of the distance. The true demand is hidden to the operator. Multiple scenarios are established to see the difference between urban settings and transferability (e.g. monocentric vs polycentric cities). Select comparisons between three learning policies are shown in Fig. 1.

Second, an experimental network is generated from 55 Public Use Microdata Areas (PUMAs) in NYC. After reviewing the roadway network, 246 links connect 55 nodes in this network shown in Fig. 2(a). The mean OD flows between zones are drawn from the RHTS while their standard deviations and covariances are randomly assumed. Single transfers are allowed between routes.

Due to the limitation on actual experiments, observed OD flows were randomly drawn from assumed true values under the assumption that they follow normal distribution but are correlated, forming a multivariate normal distribution. The covariance matrix of OD flow is also artificially created but required to be a positive semidefinite matrix to derive multivariate normal random numbers from given mean vector and covariance matrix. Three scenarios are considered: a base scenario, in which OD pairs in the RHTS having 0 values are assumed to always be 0; a scenario where 0 value OD pairs from RHTS can still have demand; and a “high variation” scenario where the covariance matrix values are increased significantly.

3 RESULTS

3.1 Example Grid Network

KGCB shows the best total covered demand (Table 1): KGCB outperforms KG on average of 27%, increasing to 33% when transfers are allowed. When correlations are ignored using KG, its average performance is even slightly worse than MAB.

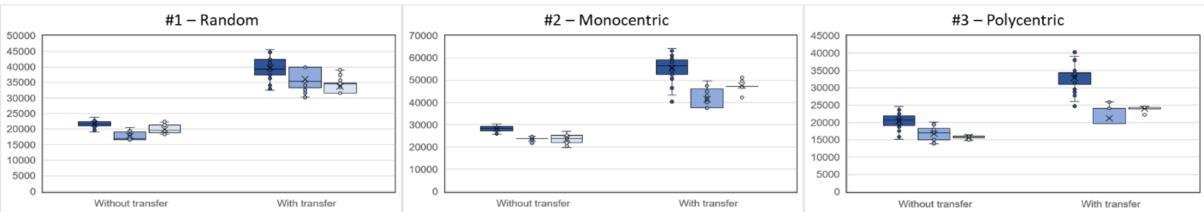


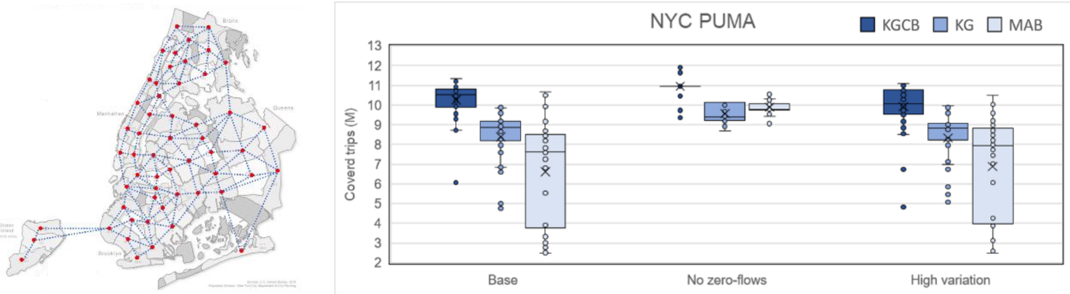
Figure 1 – *Box plots of results from example grid network*

Table 1 – *Mean of covered demand from example grid network relative to KG (%)*

Policy	#1 – Random		#2 – Monocentric		#3 – Polycentric	
	w/o transfer	w. transfer	w/o transfer	w. transfer	w/o transfer	w. transfer
KGCB	122%	110%	119%	134%	122%	155%
KG	100%	100%	100%	100%	100%	100%
MAB	112%	92%	99%	115%	95%	113%

3.2 Real-world Example: New York City

The results for the NYC example (Table 2, Fig. 2(b)) indicate MAB has much larger variances, while KGCB attains the highest mean (average of 19% higher than KG) and lowest variance.



(a) (b)

Figure 2 – *(a) Map of NYC PUMA network and (b) box plots of results from NYC case*

Table 2 – *Mean, (standard deviation), [% of KG] of covered demand from NYC case*

(unit: M trips)	#1 – Base	#2 – No zero-flows	#3 – High variation
KGCB	10.264 (0.887) [122%]	10.949 (0.391) [114%]	9.925 (1.192) [119%]
KG	8.380 (1.282) [100%]	9.564 (0.468) [100%]	8.330 (1.345) [100%]
MAB	6.618 (2.618) [79%]	9.887 (0.312) [103%]	6.877 (2.471) [83%]

4 DISCUSSION

This study proposes a new approach based on an optimal learning technique within a sequential transit route design framework. Since it sequentially extends the system by choosing an option within a given set based on the most recent knowledge updated after observing the consequence of choices made, optimal learning techniques are employed. Three learning policies, KGCB, KG, and MAB are compared, and KGCB shows the best performance represented by mean covering demand.

This artificial intelligence (AI) methodology is designed for designing systems without abundant demand data to begin with and makes use of the sequential to "grow" this knowledge base. It reduces the dependency on existing data sources and promotes active information accumulation which makes the system more responsive to gaps that could not be captured by previous data collection. Furthermore, due to its gradual expansion of the system, it can be more attractive when the project budget is divided over time and sequentially executed.

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