Markovian Dynamic Traffic Assignment: A new approach for stochastic DTA

R. de la Paz Guala[a,*], C.E. Cortés[a,b], B. Heydecker[c] and P.A. Rey[d]

[a] Civil Engineering Department, FCFM, Universidad de Chile, Santiago, Chile
ricardo.delapaz@uchile.cl, ccortes@ing.uchile.cl
[b] Instituto de Sistemas Complejos de Ingeniería (ISCI), Santiago, Chile
[c] Centre for Transport Studies UCL, London, United Kingdom
b.heydecker@ucl.ac.uk
[d] Department of Industry, Universidad Tecnológica Metropolitana, Santiago, Chile
prey@utem.cl
[*] Corresponding author

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1 INTRODUCTION

In dynamic traffic assignment (DTA) applications, it is relevant to consider the uncertainty inherent to motorist route choices as part of the DTA formulations. In particular, motorists' choices made on realistic transport networks are mostly based on the perceived costs of all routes from their origins to their destinations. In this work, we present an approach to address stochasticity in a DTA context based on nested cost operators, where motorists construct their routes taking into account the perceived costs from their current node to their destinations.

We present the Markovian dynamic traffic assignment (MDTA) model, an integration of the Markovian traffic equilibrium (MTE) by Baillon & Cominetti (2008) with the DTA modelling framework by Addison & Heydecker (1996, 1998). It addresses stochasticity in a DTA context where motorists choose their route according to their perceived costs of the remaining trip, resulting in an arc-based model instead of a route-based one. Our model allows working with overlapping routes with no assumptions of independence of their costs. We comment on the MDTA algorithm, a method that solves our model in discrete time and that respects the First In First Out (FIFO) rule and on preliminar results regarding its computational implementation.

2 The Markovian dynamic traffic assingment approach

We first address two aspects that are key to our approach.

2.1 Reasonable arcs towards a destination

From a purely stochastic point of view, every option has a positive probability of being chosen by users. In reality, it is observed that not actually all options are even considered. To address this, we adapt a *reasonability* concept.

Considering a destination node d, we say that (i,j) is a *reasonable arc towards destination* d if the minimum cost of going from node i to d is greater or equal to the minimum cost of going from node j to d. The set of reasonable arcs towards a destination node d is denoted as R^d. Our proposed definition is an adaptation of the *reasonable route* concept from Dial (1971) (originally defined over routes and origin-destination pairs instead of arcs and destinations).

2.2 Fulfilling the FIFO rule

To illustrate how FIFO rule is fulfilled, we provide a simplified description of how the approach performs the queue unloading process in a basic case scenario.

Consider an arc with two flow of motorists queueing, going to go destinations A and B, denoted as A_1 and B_1, respectively. Now, consider two new flows joining the queue (one for every destination), denoted as A_2 and B_2. If the queue unloading capacity of the arc, Q, is not overpassed by the aggregation of flows ($Q \geq A_1+B_1+A_2+b_2$), then all motorists can leave, and there are outflow rates of $A_1 + A_2$ (going to A) and $B_1 + B_2$ (going to B). Otherwise, consider that $A_1+B_1 \leq Q < A_1+B_1+A_2+b_2$, then priority is given to those motorists waiting the most (A_1 and B_1) and the residual unloading capacity is proportionally split between the motorists that arrived right after those that have priority (A_2 and B_2), then, there are outflow rates of $A_1 + (Q - A_1 - B_1)\frac{A_2}{A_2+B_2}$ (going to A) and $B_1 + (Q - A_1 - B_1)\frac{B_2}{A_2+B_2}$ (going to B).

2.3 The Markovian dynamic traffic assignment model

The MDTA model has: a demand profile, a traffic model and an arc-choice model. For each O-D pair $(o,d) \in OD$, the time-dependent demand rate function from the origin node o to the destination node d, $\mathcal{D}_{od}(\cdot)$, is exogenous. The other two parts are next further addressed.

2.3.1 The traffic model

We adapt the *deterministic punctual queueing model* to represent the traffic behaviour within each arc, considering its features referenced in Addison & Heydecker (1998).

For each destination $d \in D$, for each arc $a \in A$ and at each time $t \in [0,T]$, the inflow rate and outflow rate of arc a going to destination d at time t are denoted as $E_{ad}(t)$ and $G_{ad}(t)$, respectively. The number of motorists with destination d in a queue on arc a at time t is denoted as $L_{ad}(t)$, which we refer at as the *queue length going to* d *of* a *at* t.Then, the relationships between inflow rates, outflow rates and queue lengths, for each destination node $d \in D$ for each arc $a \in A$ at each time $t \in [\phi_a, T+\phi_a]$ can be analytically expressed as:

$$G_{ad}(t) = \begin{cases} E_{ad}(t-\phi_a), & \text{if } \sum\limits_{d' \in D} E_{ad'}(t-\phi_a) \leq Q_a \wedge \sum\limits_{d' \in D} L_{ad'}(t) = 0, \\ \dfrac{L_{ad}(t)}{\sum\limits_{d' \in D} L_{ad'}(t)} Q_a, & \text{otherwise}, \end{cases} \tag{1}$$

$$\frac{dL_{ad}}{dt} = \begin{cases} 0, & \text{if } \sum\limits_{d' \in D} E_{ad'}(t-\phi_a) \leq Q_a \wedge \sum\limits_{d' \in D} L_{ad'}(t) = 0, \\ E_{ad}(t-\phi_a) - G_{ad}(t), & \text{otherwise}. \end{cases} \tag{2}$$

Now, for each arc $a \in A$ and at each time $t \in [0,T]$, the cost of the arc a, having entered it at t, denoted as $C_a(t)$, is given by the free flow travel time of a plus the delay due to the waiting time in the queue. Analytically, this can be expressed as:

$$C_a(t) = \phi_a + \frac{\sum\limits_{d' \in D} L_{ad'}(t+\phi_a)}{Q_a}. \tag{3}$$

As FIFO rule must be satisfied, for each $d \in D$, for each arc $a \in A$, and at each time $t \in [0, T]$, as in Heydecker & Addison (2005), we set the following condition:

$$G_{ad}\left(\tau_a(t)\right) = \frac{E_{ad}\left(t\right)}{\frac{d\tau_a(t)}{dt}}, \tag{4}$$

where $\tau_a(t)$ is the exit time of a motorist that entered arc a at instant t, given by $\tau_a(t) = t + C_a(t)$.

Note that, given the arc-based construction of the model and, particularly, the arc costs, our approach is able to address overlapping routes without independence of route costs.

2.3.2 The arc-choice model

Our model is a dynamic adaptation of the static traffic assignment model embedded in the *MTE* (Baillon & Cominetti, 2008), which considers that motorists choose following a logit model that considers the expected minimum costs from the current node to their respective destinations. We also use a logit model, considering a constant and known dispersion parameter θ.

For each destination node $d \in D$, for each arc $a = (i, j) \in A$ and at each time $t \in [0, T]$, the expected minimum cost of going from i to d using arc a, entering at t, $Z_{ad}(t)$, is given by:

$$Z_{ad}(t) = C_a(t) - \frac{1}{\theta} \ln \left(\sum_{b \in A_j^+} \exp\left(-\theta Z_{bd}\left(t + C_a\left(t\right)\right)\right) \right). \tag{5}$$

Now, let us recall that only outgoing arcs a from a node i that are reasonable towards d ($a \in A_i^+ \bigcap R^d$) are assigned positive inflow rates. Additionally, at a given instant, the flow rate to be assigned from node i can come from two sources: the aggregate outflow rate of incoming arcs to i and/or demand rate being generated at i (if it is an origin).

For each destination node $d \in D$, there are two cases: (1) for each node $i \in N$ such that $(i, d) \notin OD$ (nodes that are not origins for destination d), for each arc $a = (i, j) \in A_i^+$ and at each time $t \in [0, T]$, the inflow rate of a going to d at t is given by:

$$E_{ad}(t) = \begin{cases} \frac{\exp(-\theta Z_{ad}(t))}{\sum\limits_{b \in A_i^+ \bigcap R^d} \exp\left(-\theta Z_{bd}\left(t\right)\right)} \sum\limits_{b \in A_i^-} G_{bd}(t), & \text{if } a \in R^d, \\ 0, & \text{otherwise;} \end{cases} \tag{6}$$

and (2), for each $o \in O$ such that $(o, d) \in OD$ (nodes that are origins for destination d), for each arc $a = (o, j) \in A_o^+$ and at each time $t \in [0, T]$, the inflow rate of a going to d at t is given by:

$$E_{ad}(t) = \begin{cases} \frac{\exp(-\theta Z_{ad}(t))}{\sum\limits_{b \in A_o^+ \bigcap R^d} \exp\left(-\theta Z_{bd}\left(t\right)\right)} \left(\sum\limits_{b \in A_o^-} G_{bd}(t) + \mathcal{D}_{(o,d)}(t) \right), & \text{if } a \in R^d, \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

2.4 The MDTA algorithm

To solve a discrete version of the problem, we present the MDTA algorithm. At each time increment resulting from the discretization, it runs an adaptation of Dial's algorithm (Dial, 1971), where the backward step is run first to compute the expected minimum costs from each destinations, and then the forward step is run to compute the inflow rate assignments.

The MDTa algorithm has been computationally implemented and some comments on one of the tested instances are presented in section 3.

3 Results and final remarks

We first briefly comment on some graphic results obtained from applying the MDTA algorithm on an artificial instance for the Sioux Falls network (Yang & Qiang, 2016), where, for each arc a, its free flow travel time ϕ_a and its queue unloading capacity, Q_a are known and constant. Figure 1 depicts the demand rates for each O-D pair and the evolution of the outputs (inflow rates, outflow rates, and queue lenghts) of arc 22, where $\phi_{22} = 2$ $[sec]$ and $Q_{22} = 3$ $[veh/sec]$. Some important aspects regarding the behavior of the outputs are: (1) as arc 22 is only reasonable towards node 13 and 20 (not towards 2), then there are two types of motorists traveling through the arc; (2) when the total inflow rate (sum of inflow rates going to both destinations) becomes greater than Q_{22}, then a queue forms; (3) when there is a queue, then the queue unloading happens at capacity, then the total outflow rate is equal to Q_{22} (while fulfilling FIFO rule).

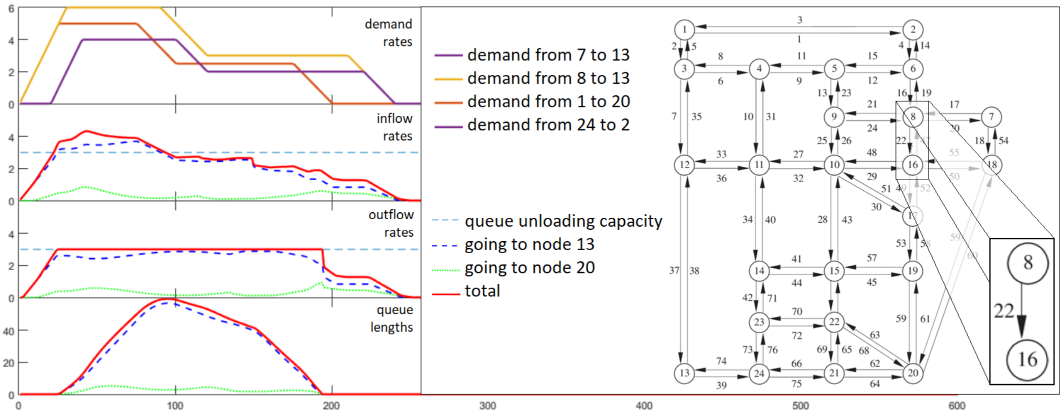

Figure 1 – *Demand rates* $[veh/sec]$ *for each O-D pair, and arc 22's inflow rates* $[veh/sec]$*, outflow rates* $[veh/sec]$*, and queue lengths* $[veh]$ *going to destinations 13 and 20*

We have provided the highlights of our Markovian approach to deal with DTA in an stochastic context. The main contribution presented in this extended abstract is the Markovian dynamic traffic assignment model, where motorists construct their route based on recursive arc choices based on the expected costs to their respective destinations while choosing only arcs that they consider to be reasonable. This model, given its arc-based formulation, allows working with overlapping routes with no asumptions on independence of route costs, avoiding the usually necesary route enumeration process that is found in literature when applying route-based formulations. We also present some results of the MDTA algorithm, a deeply elaborated method that allows to obtain a solution for the model, respecting FIFO rule, that, given its detailed dynamic programming nature, its a determinant contribution by itslef.

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