A branch-price-and-cut algorithm for a Multi-Commodity two-echelon Distribution Problem

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1 Introduction

In this presentation, we consider a *Multi-Commodity two-echelon Distribution Problem* (MC2DP) which aims to design a two-echelon distribution system composed of a set of suppliers $\mathcal{S}$, a set of distribution centers $\mathcal{D}$ and a set of customers $\mathcal{C}$. Collection operations are performed at suppliers, while delivery operations are performed at customers. More precisely, $|\mathcal{K}|$ commodities are collected from the suppliers, sent to the distribution centers for consolidation purposes (first echelon), and delivered to the customers to fulfill their requests (second echelon). For each commodity $k \in \mathcal{K}$, each supplier $i \in \mathcal{S}$ provides an amount $P_{ik} \geq 0$ of k, and each customer $j \in \mathcal{C}$ has a request $R_{jk} \geq 0$ for k. An unlimited fleet of homogeneous vehicles of capacity Q^1 is available for the collection operations. Suppliers are linked to the distribution centers by direct trips. Each distribution center owns an unlimited fleet of homogeneous vehicles of capacity Q^2 performing routes to deliver the commodities to the customers.

All vehicles can transport any set of commodities as long as their capacity is not exceeded. In addition, as in the *Commodity constrained Split Delivery Vehicle Routing Problem* (C-SDVRP) (see Archetti *et al.* (2016)), customers can be visited multiple times. However, the request for a given commodity has to be delivered in a single visit. The MC2DP aims to fulfill the customer requests not exceeding the vehicle capacities and the available commodity amounts at the suppliers and such that the overall transportation cost is minimized.

The classical 2E-CVRP considers the*one-to-many* case, where a single commodity has to be delivered from a depot to a set of customers through distribution centers by means of two levels of routing decisions. Only a few papers (Dellaert *et al.* (2021)) address a *many-to-many* setting. The connection of the routes then considers the movement of commodities from the first echelon to the second echelon. The commodities are non-substitutable. Each commodity has an origin in the first echelon and a destination in the second. The MC2DP differs by considering multiple substitutable commodities in a*many-to-many* setting. Indeed, commodities available in limited quantities have to be collected from multiple suppliers and delivered to customers according to their requests, but there is no assignment between suppliers and customers.

In addition, in the 2E-CVRP, the link between the collection and delivery echelons is commonly done by the *load synchronization* strategy (Drexl (2012)). The amount of commodity sent to each distribution center must be sufficient to serve the customers assigned to that distribution center. In the MC2DP, the same synchronization strategy is applied, but it is associated with each commodity.

The study of the MC2DP was motivated by a case study presented in Gu *et al.* (2021) for the collection and delivery of fresh agri-food products (fruits and vegetables) through a short and local supply chain. More precisely, farmers deliver their products to distribution centers managed by professional associations. From them, the products are delivered to supermarkets and canteens. In Gu *et al.* (2021), the authors proposed a heuristic sequential approach (collection first, delivery second). Here, we propose an exact algorithm based on a *branch-price-and-cut* (BPC) framework.

2 Problem formulation

We define the MC2DP on a directed weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V} = \mathcal{S} \cup \mathcal{D} \cup \mathcal{C}$ and $\mathcal{A} = (\mathcal{S} \times \mathcal{D}) \cup (\mathcal{D} \times \mathcal{S}) \cup (\mathcal{D} \times \mathcal{C}) \cup (\mathcal{C} \times \mathcal{D})$. $\mathcal{A}^1$ denotes the subset of arcs : $\mathcal{D} \times \mathcal{S}$. Each arc $(i,j) \in \mathcal{A}$ is associated with cost $C_{ij} \geq 0$. For each distribution center $o \in \mathcal{D}$, $\mathcal{R}_o$ is the set of the feasible routes that a vehicle owned by o can perform, i.e., non-empty cycles in the delivery echelon starting and ending at o such that the total amount of delivered commodities does not exceed vehicle capacity Q^2. Finally, C_r is the cost associated with route r and a^r_{jk} is a binary parameter taking value 1 if r delivers commodity k to customer j and 0 otherwise.

We introduce the following variables. For all distribution centers $o \in \mathcal{D}$ and suppliers $i \in \mathcal{S}$, integer variable x_{oi} represents the number of vehicles traversing arc $(o,i) \in \mathcal{A}^1$. For all $o \in \mathcal{D}$, $i \in \mathcal{S}$ and $k \in \mathcal{K}$, non-negative continuous variable q^k_{oi} are associated with the amount of commodity k loaded at supplier i and sent to distribution center o. Finally, for all $o \in \mathcal{D}$ and $r \in \mathcal{R}_o$, binary variable λ_r takes value 1 if route r is selected in the solution and 0 otherwise. Our extended formulation, referred to as Master Problem (MP), is as follows:

$$min \sum_{(o,i)\in\mathcal{A}^1} (C_{oi} + C_{io})x_{oi} + \sum_{o\in\mathcal{D}} \sum_{r\in\mathcal{R}_o} C_r\lambda_r \tag{1}$$

subject to:

$$\sum_{o\in\mathcal{D}} q^k_{oi} \leq P_{ik} \quad \forall i \in \mathcal{S}, \forall k \in \mathcal{K} \tag{2}$$

$$\sum_{k\in\mathcal{K}} q^k_{oi} \leq Q^1 x_{oi} \quad \forall o \in \mathcal{D}, \forall i \in \mathcal{S} \tag{3}$$

$$\sum_{o\in\mathcal{D}} \sum_{r\in\mathcal{R}_o} a^r_{jk}\lambda_r \geq 1 \quad \forall j \in \mathcal{C}, \forall k \in \mathcal{K} \; s.t. R_{jk} > 0 \tag{4}$$

$$\sum_{i\in\mathcal{S}} q^k_{io} \geq \sum_{r\in\mathcal{R}_o} \sum_{j\in\mathcal{C}} R_{jk} a^r_{jk} \lambda_r \quad \forall o \in \mathcal{D}, \forall k \in \mathcal{K} \tag{5}$$

$$x_{oi} \in \mathbb{N} \, \forall (o,i) \in \mathcal{A}^1, \quad q^k_{oi} \in \mathbb{R}_+ \, \forall (o,i) \in \mathcal{A}^1 \, \forall k \in \mathcal{K}, \lambda_r \in \{0,1\} \forall r \in \mathcal{R}_o, \forall o \in \mathcal{D}. \tag{6}$$

The objective function minimizes the total transportation cost. Constraints 2 and 3 are related to the collection echelon. Constraints 2 guarantee that the quantity of each commodity picked-up at each supplier does not exceed the maximum quantity available. Constraints 3 link the quantities collected at suppliers to the number of vehicles needed to transport them. Constraints 4 are the *covering constraints* ensuring that customer requests are fulfilled. Constraints 5 impose the load synchronization strategy linking the collection and delivery echelons at the distribution centers. Constraints 6 define the domains of the decision variables.

Table 1 – *Preliminary computational results.*

Instance				Cplex			BPC		
$\|\mathcal{S}\|$	$\|\mathcal{D}\|$	$\|\mathcal{C}\|$	$\|\mathcal{K}\|$	UB	LB	t(s)/gap(%)	UB	LB	t(s)/gap(%)
4	2	10	2	579.52	549.03	5.55%	579.52	-	0.39s
4	2	10	2	562.34	-	1441s	562.34	-	0.53s
4	2	10	2	663.52	-	2261s	663.52	-	0.32s
4	2	15	2	771.34	654.28	17.89%	742.71	-	0.96s
4	2	15	2	790.28	682.49	15.79%	784.05	-	127.33s
4	2	15	2	896.12	742.96	20.62%	893.08	-	0.52s
4	2	20	2	1063.71	807.35	31.75%	1007.04	-	8.82s
4	2	20	2	1108.31	851.34	30.18%	1065.98	1077.43	1.07%
4	2	20	2	1243.96	867.64	43.37%	1177.46	-	0.55s
4	2	25	2	1188.39	904.92	31.32%	1184.62	-	70.71s
4	2	25	2	1416.33	915.13	54.77%	1258.91	-	70.08s
4	2	25	2	1526.09	952.69	60.19%	1367.61	-	11.34s

3 A branch-price-and-cut algorithm

To solve the MP introduced in Section 2, we design a BPC algorithm, where, at each column generation iteration, we price λ_r variables. Specifically, we solve the pricing problem $\min\{\bar{C}_r \mid r \in \mathcal{R}_o, o \in \mathcal{D}\}$, where $\bar{C}_r$ is the reduced cost of λ_r, by decomposing it per distribution center. Solving $\min\{\bar{C}_r \mid r \in \mathcal{R}_o, o \in \mathcal{D}\}$ reduces to solve an *Elementary Shortest Path Problems with Resource Constraints* (ESPPRC) (see Gschwind *et al.* (2019)). We tackle the ESPPRC through a label setting dynamic programming algorithm which incorporates the *ng-path* relaxation and an implicit version of the bidirectional labeling search. Our BPC implements several classical and advanced techniques (Pessoa *et al.* (2020)) to accelerate the column generation procedure: the automatic dual pricing smoothing stabilization, a multi-phase strong branching procedure, and three heuristic approaches to solve the ESPPRC. Two of these are similar to the graph reduction heuristics proposed in Gschwind *et al.* (2019). The third one is a novel two-phase heuristic. The first phase computes a lower bound on the value of $\min\{\bar{C}_r \mid r \in \mathcal{R}_o, o \in \mathcal{D}\}$ and a set of promising customer sequences: it does so by solving the ESPPRC on a modified graph, where customers are delivered with their least requested commodity. The second phase retrieves the route to be inserted in the MP by solving the ESPPRC again on several acyclic graphs, one for each customer sequence computed in the previous phase. Finally, we look for violated valid inequalities before starting the branching phase. We consider the capacity constraints and two new families of valid inequalities based respectively on the set covering polytope and on the number partitioning problem polytope. Last, we implement a sophisticated branching strategy, considering first the variables associated with the collection level and then those associated with the arcs involved in the pricing problem. To ensure the correctness of the algorithm, we need to implement the Foster and Ryan rule (see Foster & Ryan (1976)).

4 Preliminary computational results

Table 1 shows preliminary results obtained by running our BPC algorithm on 12 small instances considered in Gu *et al.* (2021). We compare our results with those obtained by solving the compact formulation of the MC2DP presented in Gu *et al.* (2021) with the commercial solver Cplex 12.8. For both approaches, we report the upper and lower bounds returned after a time limit of one hour for Cplex and of 600 seconds for our approach. We indicate the computational time, if the instance is solved to optimality ("-"in column LB), or the optimality gap computes as $\frac{UB-LB}{LB} \cdot 100$. The BPC algorithm outperforms Cplex in terms of time and solution quality: it optimally solves all but one instances in short computation times.

As future research direction, we plan to generalize the MC2DP by considering route decisions also in the collection echelon and, hence, to adapt our BPC algorithm accordingly.

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