Service Bundle Sizing and Pricing in Ride Sourcing Services

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1 INTRODUCTION

Ride sourcing services have emerged as a viable personalized mobility service to many travelers and received significant attention from the research community as well as industries. Different types of ride sourcing services are being introduced and expected to further grow in the future. As such, in recent years, ride sourcing service operators have introduced a variety of service bundles (e.g. Uber *Ride Pass* and Lyft *All-Access Plan*) in addition to the traditional pay-per-use (PPU) strategy. Introducing bundled services can provide several merits: offering a diversity of service types (e.g. daily/weekly/monthly passes) upon market needs can better meet travelers' needs, potentially increase the market share, and reduce cost for travelers but at the same time, improving efficiency of the system operations.

To our best knowledge, a number of previous research investigated these positive impacts of introducing ride sourcing service bundles and its pricing, few have focused on formulating a service bundle design, sizing, and pricing problems. In this paper, we formulate an optimization-based ride sourcing service bundle sizing and pricing problem focused on commuting trips. While we use "commuting" trips as illustration, this can simply be replaced by total miles, minutes, or price for mobility services. Travelers' choices of bundles are represented using a multivariate probit (MVP) model as choices among different bundles are clearly correlated. The proposed formulation maximizes total revenue and determines sizes and prices of the service bundles for commuting trips (i.e., same trip characteristics). The proposed model is a nonlinear program (NLP) and KKT is applied to find optimal solution.

2 METHODOLOGY

Let $\mathbb{K} = \{1, .., k, ..., K\}$ be an ordered set of available service bundles based on the number of trips it can provide (i.e. bundle sizing), $\Psi = \{\psi_1, ..., \psi_k, ..., \psi_K\}$ with corresponding prices $\mathbb{C} = \{c_1, ..., c_k, ..., c_K\}$. It is noted that $k = 1$ with service bundle size of 1 $\psi_1 = 1$ represents PPU pricing at c_1.

2.1 Commuting Trip Bundle Demand Specification based on MVP Model

The utility for commuter n who chooses monthly commuting bundle k consists of deterministic part $V_{nk}(\psi_k, c_k)$ and stochastic part (i.e. error term) ε_{nk} with mean of zero and covariance matrix Ω. Deterministic utility is defined as a linear combination of bundle size, ψ and the price, c. Then the overall utility can be defined as:

$$\begin{aligned} U_k(\psi_k, c_k) &= V_k(\psi_k, c_k) + \varepsilon_k \\ &= \beta_k + \beta_c c_k + \beta_\psi \psi_k + \varepsilon_k \end{aligned} \tag{1}$$

where β_k represents constant term for bundle k, and β_c, β_ψ represent weights between cost and size of a bundle.

According to MVP (Train, 2009), the probability for a commuter choosing bundle k is defined as below.

$$p_k(\psi_k, c_k) = \int_{-\infty}^{V_k(\psi_k,c_k)-V_0(\psi_0,c_0)} \cdots \int_{-\infty}^{V_k(\psi_k,c_k)-V_K(\psi_K,c_K)} \frac{\exp\{-\frac{1}{2}(\eta_k)(\Omega_k)^{-1}\eta_k\}}{(2\pi)^{\frac{K-1}{2}}|\Omega_k|^{\frac{1}{2}}} d_{\varepsilon_K-\varepsilon_k} \cdots d_{\varepsilon_0-\varepsilon_k} \tag{2}$$

As these probabilities are not in a closed form and this integration is difficult, we rely on GHK simulation algorithm (Cappellari & Jenkins, 2003) which enables computation of probabilities of choosing each choice given an observation data set. GHK simulation randomly chooses truncated observation ν from normal distribution. Then, the probability of choosing choice k is as follows:

$$p_k(\psi_k, c_k) = \frac{1}{|\mathcal{T}|} \sum_{\tau\in\mathcal{T}} \prod_{\forall k'\in\mathbb{K},\, k'\neq k} \Phi\left(\frac{\beta_k - \beta_{k'} + \beta_c(c_k - c_{k'}) + \beta_\psi(\psi_k - \psi_{k'}) - \sum_{\xi=0}^{k'} l_{k'\xi}\cdot \nu_\xi^\tau}{l_{k'k'}}\right) \tag{3}$$

where $\Phi(z)$ stands for CDF of standard normal distribution, $l_{k'\xi}$, $l_{k'k'}$ are extracted by lower triangular matrix $\mathcal{L}_k$ by Cholesky decomposition of reduced variance-covariance matrix Ω_k derived from variance-covariance matrix $\Omega_\mathbb{K}$. Equation (3) provides the likelihood of choices given price and the size of each bundle k.

2.2 Service Bundle Size and Pricing Model

With a demand expression that provides probabilities of choosing bundle k given its price and the size, the service bundle sizing and pricing model is formulated as below.

$$\begin{aligned} \max_{\Psi,\mathbb{C}}\ Z &= \sum_{\forall k\in\mathbb{K}} c_k \cdot d_k(\psi_k, c_k) \\ &= d_\mathbb{K} \sum_{\forall k\in\mathbb{K}} c_k \cdot p_k(\psi_k, c_k) \\ &= \frac{d_\mathbb{K}}{|\mathcal{T}|} \sum_{\forall k\in\mathbb{K}} c_k \sum_{\tau\in\mathcal{T}} \prod_{\forall k'\in\mathbb{K},\, k'\neq k} \Phi\left(\frac{\beta_k - \beta_{k'} + \beta_c(c_k - c_{k'}) + \beta_\psi(\psi_k - \psi_{k'}) - \sum_{\xi=0}^{k'} l_{k'\xi}\cdot \nu_\xi^\tau}{l_{k'k'}}\right) \end{aligned} \tag{4}$$

$$\text{s.t.:} \quad \max_{\forall k^\beth\in\mathbb{K},\psi_{k^\beth}>\psi_k} \{c_1, c_{k^\beth} - (\psi_{k^\beth} - \psi_k)c_1\} \leq c_k, \forall k \in \mathbb{K} \tag{5}$$

$$\min_{\forall k^\beth\in\mathbb{K},\psi_{k^\beth}<\psi_k} \{\psi_k c_1, c_{k^\beth} + (\psi_k - \psi_{k^\beth})c_1\} \geq c_k, \forall k \in \mathbb{K} \tag{6}$$

$$c_k \in (0, c_k^+], \psi_k \in (0, \psi_k^+], \forall k \in \mathbb{K} \tag{7}$$

$$\psi_k \neq \psi_{k'}, \forall k, k' \in \mathbb{K}, k \neq k' \tag{8}$$

Objective is to maximize the total revenue Z across offered bundles summation of all bundle pricing $c_k \in \mathbb{C}$ times corresponding quantity of subscribing respondents $d_k(\psi_k, c_k) \in d_\mathbb{K}$ incorporated with bundle choice probability Equation (3) depicted in Equation (4) where bundle price c_k and size ψ_k are only variable vectors. The bundle price c_k lower bound in inequality Constraint (5) guarantees the price difference between bundle $k^{\urcorner}$ and k, for the case bundle size of $k^{\urcorner}$ larger than bundle k, would be less or equal to these two bundle passes' difference pay by PPU because of possible discount rate and total subscription price is no less than PPU by default. The inequality Constraint (6) guarantees the bundle price c_k upper bound offered by the charge discrepancy between bundle $k^{\sqsupset}$ and k, with condition bundle size of $k^{\sqsupset}$ less than k, is less or equal to the difference bundle trips pay by PPU with discount rate and bundle price is intuitively cheaper than every time pay by PPU attached to these bundle trips. The Constraint (7) guarantees the bundle price to be positive and within a pricing range based on price per trip by empirical data and bundle size is bounded by travelers' commuting needs where c_k^+ and ψ_k^+ stand for bundle k price and size designs' upper limit. The last Constraint (8) prevents the size of different bundle being the same which offers diversity for bundle choices better meet travelers' commuting needs.

2.3 Solution Method: Tocher Approximation and KKT

The proposed formulation of Commuter Bundle Sizing and Pricing problem is a nonlinear program. Here we use an approximate objective function to be in a convex form and apply KKT. The objective function (4) with price and size constraints from (5) through (8) are still very difficult to approach as CDF of standard normal distributions in the objective plugged with variables c_k and ψ_k. The Tocher approximation (Tocher, 1967, Choudhury, 2014, Yerukala & Boiroju, 2015) further simplifies the objective maximization function shown in Equation (9) with $k = \sqrt{\frac{2}{\pi}}$.

$$\max_{\Psi,\mathbb{C}} Z = \frac{d_{\mathbb{K}}}{|\mathcal{T}|} \sum_{\forall k \in \mathbb{K}} c_k \sum_{\tau \in \mathcal{T}} \prod_{\forall k' \in \mathbb{K},\, k' \neq k} \frac{e^{2k \frac{\beta_k - \beta_{k'} + \beta_c (c_k - c_{k'}) + \beta_\psi (\psi_k - \psi_{k'}) - \sum_{\xi=0}^{k'} l_{k'\xi} \cdot \nu_\xi^\tau}{l_{k'k'}}}}{1 + e^{2k \frac{\beta_k - \beta_{k'} + \beta_c (c_k - c_{k'}) + \beta_\psi (\psi_k - \psi_{k'}) - \sum_{\xi=0}^{k'} l_{k'\xi} \cdot \nu_\xi^\tau}{l_{k'k'}}}} \tag{9}$$

With the Tocher approximation, the objective function in Equation (9) is concave then this optimization problem has global maximum. As such we can apply the KKT condition to find the optimum bounded by constraints. The vectors of all first order conditions can be enumerated by $\triangledown L_{\mathbb{K}} = [\frac{\partial L_{\mathbb{K}}}{\partial c_k}, \frac{\partial L_{\mathbb{K}}}{\partial \psi_k}]^T = 0$ with endogenous variables c_k and ψ_k with Lagrangian function $L_{\mathbb{K}}$ integrated with the objective Z. The complementary slackness vectors functions can also be derived by equality sets respectively. Then the global optimum could be reached by KKT if the objective function proved convexity with linear constraints for bundle size and pricing.

3 CASE STUDY

We develop a case study that determines a monthly commuter travel bundles. One thousand travelers are generated with their intended commuting days (between 1 to 20). Travelers are assumed to choose bundle k, based on their intended commuting days. Given two generated scenarios of bundle sets (i.e., two sets of sizes and prices), travelers' choose one bundle k based on log-normal probabilities given price and size.

The generated sample is used for the MVP model estimation for constants β_k, coefficients β_c and β_ψ by maximum likelihood estimation (MLE) and the result is presented in Table 1. The estimated parameters indicate all constants positive and respondents are sensitive to bundle price changing and discouraged by increment of price, also demonstrating at close bundle price level subscribers are prone to attach to bundles with more passes. Further the corresponding scaled covariance matrix $\Omega_{\mathbb{K}}$ can be derived where all diagonals parameters are scaled to be one.

With these parameters, results of the bundle sizing and pricing model is presented in Table 2. The computation requires limited computing time. Bundle $\mathbb{K}$ is fixed with five bundle choices

		β_0	β_1	β_2	β_3	β_4	β_c	β_ψ
	Estimate	0.63	1.31	1.98	1.72	1.25	-0.030	0.32
MVP model	Std. Error	0.17	0.34	0.60	0.44	0.25	0.006	0.07
	p-value	0.0061	0.0081	0.0131	0.0693	0.0230	0.0000	0.0000

Table 1 – *Bundle Choice MVP Model Estimation*

including PPU by empirical ride sourcing services bundle design data having $\mathbb{K} = \{0, 1, 2, 3, 4\}$ where each bundle's size and price are solved optimal. The result shows the bundle size optimum to be reasonable as the bundle size diversities and spreads appropriately to better satisfy a variety of travelers' commuting trips. Solved by bundle subscription price, the corresponding optimal trip per price for each bundle is also rational as travelers are usually more sensitive to price per trip and all solved bundle price per trip offers sound discount making the travel fair per trip cheaper than standard price (i.e. PPU price) meanwhile maximizing stakeholder's revenue.

Optimal bundle size	PPU	4.18-day bundle	9.87-day bundle	17.04-day bundle	Unlimited (20-day bundle)
Optimal bundle price	$12	$49.41	$114.69	$196.13	$227.8
Price per trip	$12	$11.82	$11.62	$11.51	$11.39
Discount	-	1.5%	3.2%	4.1%	5.1%

Table 2 – *Optimum Bundle Size and Price by KKT*

4 Conclusion

In this paper, we formulate a commuting trip service bundle sizing and pricing problem based on travelers' choices of bundles based on MVP model. MVP model accounts for traveler choices that are correlated among different bundle choices. The proposed formulation is able to generate reasonable outcomes for bundle sizing and prices and the computation is efficient. It is noted that while we have illustrated this as "commuting trips", these can simply be extended to total miles, minutes, or prices of mobility service bundles it covers. The next potential step is characterizing trips (e.g., Origin-Destination, time, etc.). Currently in this approach, all trips are represented as days (commuting trips) or can be extended as miles, minutes, or prices. However, the ride sourcing operators are highly constrained by trip requirements due to spatial-temporal mismatch of fleet and demand as well as vehicle relocations. Then, this bundle sizing and pricing can be further integrated with fleet management and relocation operations.

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