Branch-and-Price for the electric vehicle charge scheduling problem with flexible service operations

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1 Introduction

Increasing societal and political environmental awareness, resulting from climate change, as well as local and global emission problems, call for a paradigm change towards sustainable transportation systems. Herein, electric commercial vehicles (ECVs) are seen as a viable option that promise up to 20% reduction in life-cycle greenhouse-gas emissions compared to internal combustion engine vehicles (ICEVs) while also lowering operational costs.

However, large-scale fleet electrification raises a new and unresolved planning problem: ECVs require time-intensive charging which needs to be scheduled efficiently to guarantee the cost competitiveness of electrified fleets. Specifically, operators face the following challenges when scheduling charging operations: first, the number of chargers available remains limited by grid capacity constraints and high investment costs. Second, suboptimal charging patterns with deep (dis-)charging cycles accelerate battery degradation and thus require earlier battery replacements, which affect the cost-competitiveness of ECVs (cf. Taefi, 2016). Third, energy suppliers often bill industrial customers according to time-of-use (TOU) energy tariffs, i.e., they charge varying electricity prices depending on the time of consumption. With such pricing schemes, it can be worthwhile to incur accelerated battery degradation by sub-optimal charging patterns if the cost savings obtained through off-peak energy prices outweigh degradation related costs (cf. Pelletier *et al.*, 2018). Fourth, operators who want to utilize this trade-off need to consider an accurate model of the (non-linear) recharging process in their planning problem, because the error caused by over or under-estimating the charging rate may superpose attainable cost savings (Montoya *et al.*, 2017, Pelletier *et al.*, 2018). Fifth, suboptimal service schedules may limit the impact of charge-scheduling, such that operators who wish to remain cost-competitive should design vehicle schedules with charge schedules in mind and vice versa.

Addressing this joint planning problem requires an integrated planning approach which combines vehicle scheduling problems (VSPs) with charge-scheduling. So far, publications dealing with VSPs have either focused on conventional vehicles and did not account for charging-related concerns, such as battery health, station capacity, and variable energy prices (Adler & Mirchandani, 2016, Yao *et al.*, 2020, Parmentier *et al.*, 2021), or remain computationally intractable for problem sizes relevant in practice (van Kooten Niekerk *et al.*, 2017). Publications focusing on (depot) charge-scheduling problems have been limited in a similar fashion, either assuming a simplified model of the charging process (Sassi & Oulamara, 2014, 2016) or proposing algorithms that do not scale to problem sizes encountered in practice (Pelletier *et al.*, 2018).

We close this research gap by developing an exact branch and price (B&P) algorithm for an integrated charge and service operation scheduling problem, considering battery degradation, TOU energy prices, limited availability of charging infrastructure, and non-linear battery behavior. An efficient branching rule, a state-of-the-art primal heuristic, and a novel label-setting algorithm based on a continuous label representation with set-based dominance rules allow to solve instances with fleet sizes of 68 vehicles and planning horizons of five days within one hour. Our case study shows that integrated charge and service operation scheduling lowers the amount of charging infrastructure required by up to 57% and yields operational cost savings of up to 5% in a city logistics context.

2 Problem Setting

We consider an electrified fleet of vehicles $\mathcal{K}$ that needs to perform a set of *operations* over a given planning horizon, e.g., delivery tours in the context of (city) logistics or servicing customer requests in passenger transportation. We assume that the assignment of operations to vehicles is fixed but that servicing an operation can be shifted in time subject to an operation-specific time window. Vehicles can be recharged using a set of (heterogeneous) charging stations located at the depot. For this purpose, each station $f \in \mathcal{F}$ is equipped with C_f chargers that allow for parallel charging. Charging incurs costs according to the battery degradation caused and the energy price at the time of charging. In this problem setting, an operator aims to find a cost-minimal schedule of charging- and service-operations for each vehicle $k \in \mathcal{K}$ such that i) battery capacity constraints are respected, ii) each tour is serviced by its assigned vehicle, iii) departure time windows are met, and iv) charger capacity is not violated.

To model this optimization problem, we discretize the planning horizon into a set of equidistant periods $p \in \mathcal{P}$ such that charger to vehicle allocation and energy prices remain fixed within each period. To avoid charging more energy than necessary, we allow partial charging operations independent of the time discretization, such that charging may be started and interrupted at any point in time. We model non-linear charging behavior with charger-specific recharging functions which capture a vehicle's state of charge (SoC)-evolution over time when charging with an initially empty battery and quantify the charging cost attributed to battery deterioration based on a so-called *wear density function* (cf. Han *et al.*, 2014). This function is non-linear such that we consider its piecewise linear approximation in our planning problem.

We propose a set-covering based integer programming formulation where each column models a feasible vehicle schedule $\omega \in \mathcal{A}_k$ for each vehicle $k \in \mathcal{K}$ to express our planning problem:

$$z_{MP} = \min \sum_{k\in\mathcal{K}} \sum_{\omega\in\mathcal{A}_k} x^k_\omega c(\omega) \tag{1a}$$

$$\sum_{k\in\mathcal{K}} \sum_{\omega\in\mathcal{A}_k} x^k_\omega \cdot \boldsymbol{A}^\omega_{p,f} \leq C_f \qquad f \in \mathcal{F}, p \in \mathcal{P} \tag{1b}$$

$$\sum_{\omega\in\mathcal{A}_k} x^k_\omega \geq 1 \qquad k \in \mathcal{K} \tag{1c}$$

$$x^k_\omega \in \{0,1\} \qquad \omega \in \mathcal{A}_k \tag{1d}$$

Here, $\boldsymbol{A}^\omega$ is a binary matrix that indicates whether a vehicle uses period p to charge at charger f ($\boldsymbol{A}^\omega = 1$) or remains idle, charges at a different charger $f' \in \mathcal{F} \setminus \{f\}$, or provides service ($\boldsymbol{A}^\omega = 0$). Binary variables x^k_ω indicate the inclusion of a schedule in the final solution ($x^k_\omega = 1$). Objective (1a) minimizes overall scheduling costs. *Linking* Constraints (1b) enforce charger capacity limitations, while *convexity* Constraints (1c) ensure that a schedule is picked for each vehicle. Finally, Constraints (1d) state our decision variables' domain.

3 Methodology

We propose a B&P-based approach to solve IP 1: we solve an LP relaxation of IP 1, the so-called *master problem*, using a column-generation procedure at each node of the branch and bound (B&B) tree. Specifically, we limit IP 1 to a small subset of schedules $\mathcal{A}$, which we generate iteratively. At each iteration, we solve a linear relaxation of IP 1 to then generate new schedules $\omega \in \mathcal{A}_k$ based on the dual variables of Constraints (1b) and (1c), denoted $\pi_{p,f}^{(1b)}$ and $\pi_k^{(1c)}$ respectively, in the so-called *pricing problem*:

$$\min_{k\in\mathcal{K},\omega\in\mathcal{A}_k} c(\omega) - \pi_k^{(1c)} - \sum_{p\in\mathcal{P}}\sum_{f\in\mathcal{F}} y_{\omega,p,f}\cdot\pi_{p,f}^{(1b)}. \tag{2}$$

The procedure terminates when (2) is positive, i.e., when the basis of IP 1 is optimal. We express our pricing problem as a shortest path problem with resource constraints (SPPRC) over a time-expanded network, which we solve by using a problem-specific labeling algorithm. Here, a canonical label respresentation which tracks each resource as a scalar value remains computationally intractable as each charging decision constitutes a trade-off between cost and SoC, such that time-continuous recharging requires to create an unbounded number of labels at each charging station. We address this issue with a novel label representation which captures charging trade-offs in so-called cost profiles. These state the maximum SoC reachable at the end of a (partial) path ρ when spending a total of c along ρ, potentially replenishing additional energy at f. This allows *delaying* the decision on how much to charge at f until another station or the network sink is reached. Here, the trade-off becomes explicit and a finite subset of non-dominated charging decisions at f can be identified. A novel set-based dominance criterion ensures that this function-based label representation remains computationally tractable.

As column generation deals with the Linear Program (LP) relaxation of the master problem (MP) it may not terminate with an integral solution. In such cases, we resort to B&B to establish integrality. Specifically, we observe that in each fractional solution of the restricted master problem (RMP) at least two schedules compete for a charger that is already at capacity and hence create branches based on charger allocation. We enforce any added cuts in our pricing problems to avoid additional dual variables.

We present various speedup techniques to further accelerate our B&P algorithm: we use a primal heuristic to quickly find upper bounds, thus speeding up the solution procedure by allowing to prune nodes of the B&B tree early. To this end, we use a diving heuristic that explores an auxiliary branch-and-bound tree, which branches on the variables of the extensive formulation x_ω^k, in a depth-first fashion. We rely on strong branching to boost the success rate and solution quality of the diving algorithm. Here, we consider only a subset of columns, which we select according to a roulette wheel criterion based on *dissimilarity* and *quality*, to speed up the procedure.

4 Numerical Experiments

Our computational study comprises two experiments: first, we validate the correctness, investigate the performance, and analyze the scalability of our B&P algorithm. For this purpose, we benchmark our algorithm against an equivalent mixed integer program (MIP) based formulation on a set of small, randomly generated, instances. We further test our algorithm on a set of larger instances, where we additionally vary several instance parameters. Second, we assess the impact of integrated service and charging operation scheduling in a potential real-world scenario for an electrified fleet. Specifically, we analyze the overall benefit of flexible service operations, the impact of schedule flexibility on the amount of charging infrastructure required, and the impact of energy price distribution on the cost savings obtainable through integrated charge and service operation scheduling. Table 1 shows the results of our benchmark on the small instances. As

Table 1 – *Results of our experiments on small instances.*

Algorithm	Avg. t[s]	Avg. obj	Avg. LB	Avg. #nodes	#optimal	#without solution	#total
Branch-and-Price	0.85	396.56	396.56	4.98	50	0	50
MIP	3600.00	397.46	114.74	2183022.26	1	2	50

Note. All instances were solved on a `Intel(R) i9-9900, 3.1 GHz CPU` with `16 GB` of `RAM` using a single thread.

can be seen, our algorithm outperforms the MIP formulation on all instance types. Our computational study on large instances shows that our algorithm scales to a fleet size of 68 vehicles and manages to solve instances spanning up to 5 days within the hour, allowing for day-ahead planning in practice.

Concerning our managerial study, we disseminate the following findings:

1. flexible service operations have an overall positive impact on the objective value. Specifically, higher schedule flexibility allows utilizing periods with energy prices cheap enough to outweigh additional battery degradation costs caused by cycling the battery at higher SoC levels, such that energy costs decrease and battery degradation costs increase.

2. integrated planning of charge and service operations reduces the amount of charging infrastructure required for fleet operation but shows decreasing marginal benefits: time windows of one hour reduce the number of chargers required by 14%, three hours by 43%, and six hours by 57%.

3. the mean energy price offsets total cost. Concerning energy price variance, integrated scheduling of charge and service operations provides the largest savings in scenarios with highly varying energy prices. Specifically, doubling and quadrupling the energy price variance roughly doubles and triples relative savings.

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