Workload equity for a dynamic multi-period routing problem in the context of medical transportation

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1 INTRODUCTION

The studied problem concerns the non-emergency transportation of patients. Some disabled or old patients cannot go back and forth to the hospital by their own to get cares. In this context, transport companies are involved to provide medical transportation of patients. In this work, we consider a single hospital and we assume that a single company is involved with a limited number of drivers. Transportation requests are daily revealed on a time horizon of several days.

In practice, there exists service provider platforms to help managing the schedule of vehicles routes and the assignment of routes to drivers. Generally, these platforms use basic algorithms based on assignment rules (e.g. the FIFO rule "First In First Out"). These rules can be inefficient for large scale transportation systems or when the demand varies strongly. Another drawback of these rules is that they create inequities between drivers in terms of route cost (duration or distance) and/or painfulness of work. In this work we consider the latter.

Equity has gained interest in the VRP literature in the last two decades Matl *et al.* (2018), Matl *et al.* (2019). The literature mainly focuses on single period problems. For most papers where it is considered a larger horizon, the problem is often defined as an extension of the Periodic Vehicle Routing Problem (PVRP). In this case, the demand is assumed to be completely known in advance, which is not realistic in many contexts. In multi-period problems, the total workload can be balanced among periods, delivery points or drivers. The few papers dealing with equity in a dynamic multi-period context usually consider equity among periods or delivery points and do not involve individual workloads of drivers Wen *et al.* (2010), Khorsi *et al.* (2020).

The originality of our problem is to consider equity for drivers on the "medium" term (typically one month), with requests dynamically revealed. Compared to standard models, it gives the opportunity to accept unbalanced routes at some periods (days), as long as equity is preserved on the whole horizon. In this work, we investigate different solution approaches and evaluate computationally how addressing equity this way allows limiting its impact on routing costs.

2 PROBLEM DESCRIPTION

The studied problem is defined on a large horizon $\mathcal{T} = \{1, \dots, T\}$ of T periods. A set $\mathcal{M} = \{m_1, \dots, m_K\}$ of K drivers is available. We assume that they work every day with identical vehicles of capacity Q. We distinguish two types of requests: inbound and outbound requests. Inbound requests consist in transporting patients from their home to the hospital, while outbound requests consist in transporting patients from the hospital to their home. With each request i are associated a release date r_i (time at which the patient is available at the pickup location), a due date d_i (latest time at which the patient can be dropped off at the delivery location), and a service time s_i at the customer location. Service time is also spent at the hospital, when inbound customers are dropped-off and when outbound customers are picked-up. In what follows, terms request, patient and customer will be used indifferently, and the same for terms period and day.

The problem is dynamic in the sense that requests are revealed day by day but requests of a day t are all known before starting the day. So, even though the problem is dynamic, each daily routing problem is static. This daily problem is defined as a Multi-Trip Vehicle Routing Problem with Mixed Pickup and Delivery, and Release and Due dates (MTMPD-RD) Agius *et al.* (2021).

On a given day, daily routes are first computed and then assigned to the drivers. The objective is to obtain a good equity between drivers at the end of the horizon, while not deviating too much from the travel cost that would have been obtained if equity was not considered.

3 MEASURE OF EQUITY AND SOLUTION METHOD

In our model, equity is required with regards to request painfulness. We measure the painfulness of a request i on period t as a value π_i^t defined taking into account the characteristics of patients: their age, their mobility and the type of residential building where they live. The more a request is painful, the higher π_i^t is. This metric is said constant-sum as for any feasible solution, the sum of painfulness assigned to drivers remains constant.

The total painfulness of period t is denoted $\Pi^t = \sum_{i=1}^{n} \pi_i^t$ and painfulness assigned to driver m_k on period t is Π_k^t. When solving period t, Π_k^t is unknown while the cumulative painfulness assigned to driver m_k up to period $t-1$ ($\sum_{t'=1}^{t-1} \Pi_k^{t'}$) is known and can be used.

To deal with the multi-objective nature of our problem, instead of considering two explicit objectives, we consider cost minimization as the single objective, and equity is ensured by an assignment strategy (we call it RFES for Routing First Equity Second) possibly combined with equity constraints in the daily routing problem. We propose 4 solution approaches. The first approach consists in using the RFES strategy without introducing any equity constraint in the routing. The three other approaches integrate equity constraints.

The RFES strategy is applied when vehicle routes are known in period t. As drivers are supposed identical, routes can indifferently be assigned to any driver. We assign the most painful route to the driver with the least cumulative painfulness up to period $t-1$, the second most painful route to the driver with second least cumulative painfulness, and so on to balance the cumulative painfulness up to period t.

The different approaches for the routing problem are briefly described below:

1. Routing First Equity Second (RFES): no equity constraints.

2. Single Period Constant Equity (SPCE): this approach is equivalent to RFES with equity constraints. For each driver m_k in a given period t, we limit the painfulness of the route assigned to m_k. The limit is independent of the driver:

$$\Pi_k^t \leq \alpha \times \frac{\Pi^t}{K}, \texttt{with } \alpha > 1 \tag{1}$$

3. Multi-Period Constant Equity (MPCE): This approach is equivalent to RFES with equity constraints depending on the driver. Again, for each driver m_k in a given period t, we limit the painfulness of the route assigned to m_k, but here the limit is driver-dependent. It takes

into account the cumulative painfulness assigned to drivers up to period $t-1$:

$$\sum_{t'=1}^{t} \Pi_k^{t'} \leq \alpha \times \sum_{t'=1}^{t} \frac{\Pi^{t'}}{K}, \texttt{with } \alpha > 1 \tag{2}$$

4. Multi-Period Adapted Equity (MPAE): This approach is similar to MPCE with a decreasing limit on painfulness over the time horizon (α^t decreasing uniformly from α_s at $t=1$ to α_e at $t=T$).

$$\sum_{t'=1}^{t} \Pi_k^{t'} \leq \alpha^t \times \sum_{t'=1}^{t} \frac{\Pi^{t'}}{K}, \alpha^t = \alpha_s - (t-1) \times \frac{\alpha_s - \alpha_e}{T-1} \tag{3}$$

Parameters α, α^t, α^s are defined to guarantee a given level of equity. Given a solution (of the multi-period problem), equity can be measured with several indicators, we choose the range as main indicator (difference between the drivers with the most and the least cumulative painfulness).

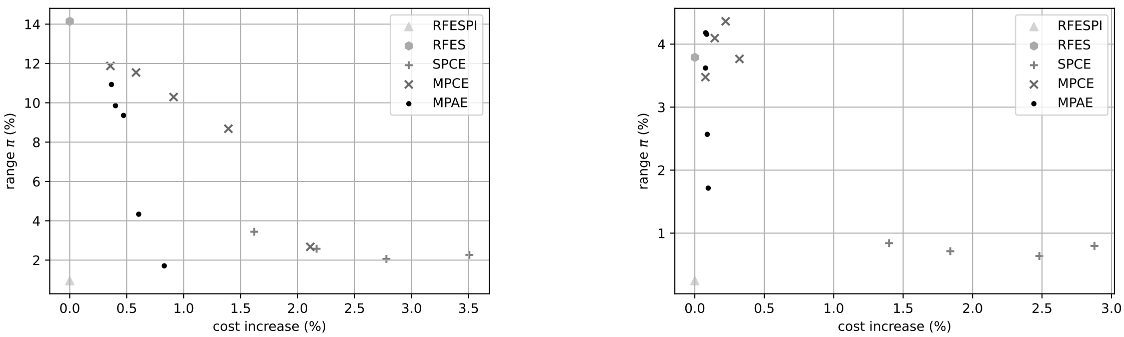
For the purpose of evaluation, we also propose another approach that we call Routing First Equity Second with Perfect Information (RFESPI) which consists in solving all single-period problems first, and then assign routes to drivers for all periods minimizing the range. In this approach, we assume that all requests of all days are known in advance, which is not realistic in the practice. This approach provides the minimal routing cost and, even if it does not mathematically guarantee a lower bound on the range when the cost is minimized, it should give a good approximation on the best range that could be achieved.

The periodic problem is solved with a specific Branch-and-Price algorithm. Its complete study is not the object of this work, instead we focus on the impact of adding the equity constraints (mentioned above) on the column generation algorithm. Adding equity constraints enforce to add a specific resource in the label definition of the pricing problem. We call the resource π. Each time a customer i is visited, π is updated $\pi \leftarrow \pi + \pi_i^t$. For SPCE, the management of resource π is easy as the limit is the same for all routes. However, for MPCE and MPAE, the limit of painfulness varies between drivers within a same period.

The intuitive way to tackle this difficulty would be to introduce a different pricing problem for each driver. Instead, we modify the Master Problem. Drivers have individual limits of painfulness but some might be equal. Hence drivers can be grouped into L groups. Each group g_l is composed of K_l drivers with the same limit π_{max}^l ($l \in \{1,\ldots,L\}$) and $\pi_{max}^{l_1} < \pi_{max}^{l_2}$ if $l_1 < l_2$. Only the drivers of group g_L are allowed to have a route more painful than π_{max}^{L-1} so, at most K_L routes can exceed π_{max}^{L-1}. In the same principle, at most $K_{L-1} + K_L$ routes can exceed π_{max}^{L-2} and so on, which gives $L-1$ new constraints. In the pricing problem, for each groups g_l ($l > 1$), the new associated dual variable λ_0^l is not counted initially and counted when the level of painfulness of the current route exceeds the limit π_{max}^{l-1}.

4 NUMERICAL EXPERIMENTS

Experiments are conducted on a benchmark of realistic instances extracted from the city of Aix-en-Provence, France. The painfulness of a request π_i^t is a value selected uniformly in the set $\{0,1,2,4,8\}$. 50% of the requests are inbound, resp. outbound. We generated 10 instances. Each instance is composed of 20 periods of 50 requests and corresponds to a time horizon of a month (assuming that drivers work 5 days a week). From those 10 instances we deduced 10 instances of 5 days (one working week). For all instances, we apply the 5 approaches mentioned above with different parameters for approaches involving equity constraints: SPCE and MPCE for all $\alpha \in \{1.1, 1.08, 1.06, 1.04, 1.02, 1.01\}$ and MPAE with $\alpha_s = 1.1$ and for all $\alpha_e \in \{1.08, 1.06, 1.04, 1.02, 1.01\}$.



(a) *Experiments with* $T = 5$ (b) *Experiments with* $T = 20$

Figure 1 – *Results of experiments on instances with* $T = 5$ *and* $T = 20$ *periods*

Figure 1 shows computational results for instances with 5 and 20 periods. Each point corresponds to an approach (combined with a parameter value α for SPCE and MPCE, or α_e for MPAE) and is located according to the average value of cost and equity on the 10 instances. The x-axis corresponds to the average increase (in %) of travel cost against the best cost (obtained by RFES and RFESPI). The y-axis corresponds to the average range expressed as a percentage of the average painfulness.

In addition, Table 1 shows the number of times a feasible solution was found by the different methods. Indeed, enforcing the equity too strictly sometimes prevent from finding a feasible solution (in at least one period of the horizon).

Table 1 – *Feasibility of different approaches*

Approach	RFESPI	RFES	SPCE						MPCE						MPAE				
α/α_e	-	-	1.1	1.08	1.06	1.04	1.02	1.01	1.1	1.08	1.06	1.04	1.02	1.01	1.08	1.06	1.04	1.02	1.01
#feas inst for $T = 5$	10	10	10	10	10	5	0	0	10	10	10	8	1	0	10	10	10	10	9
#feas inst for $T = 20$	10	10	10	10	9	2	0	0	10	10	10	8	0	0	10	10	10	10	10

These results will largely be discussed at the conference.

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