# An evaluation of the fairness of railway timetable rescheduling in the presence of competition between train operators

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## 1 INTRODUCTION

Effective real-time management of railway traffic is crucial to delivering good railway performance. In particular, making changes to the timetable in response to an initial delay can help to reduce the amount of additional delay caused to other trains as a result of the initial incident. This practice is known as *timetable rescheduling.* The Train Timetable Rescheduling Problem (TTRP) (Cacchiani *et al.*, 2014) can be solved in order to determine the optimal way to reschedule the timetable. A large number of different TTRP problem variants, models, objective functions and solution methods have been studied.

However, the implications for TTRP models of economic competition between railway operators has not been considered widely, see (Luan *et al.*, 2017). In recent decades, different forms of competition have been introduced in several European railway systems, such as those of Germany, Great Britain and Sweden (IBM, 2011). Where trains are operated by more than one different company over the same tracks, timetable rescheduling has the potential to impact these operators unequally. In order to be perceived as fair, a TTRP model must not systematically favour some operators over others. A perception of unfairness would be a serious barrier to the practical deployment of TTRP models in competitive railway systems. Therefore, it is essential that the fairness characteristics of such models are understood. This study investigates the fairness of solutions obtained from solving the TTRP.

This paper is organised as follows. We first describe our methodology by defining our notions of fairness and efficiency and how to evaluate them. In our results section, we present an analysis of the fairness of efficiency-maximising TTRP solutions. This is supplemented by an analysis of the interactions between pairs of operators, where we also consider the fairness-efficiency trade-off.

# 2 METHODOLOGY

## 2.1 Efficiency

Our measure of the overall system efficiency was developed with Network Rail for our TTRP system (see Reynolds *et al.* (2020)). It is designed to model the utility of Network Rail, which can be seen as the central decision maker for rescheduling decisions. It is designed to take into account the overall quality of service provided to passengers.

Given a feasible solution $x$ to a given instance, the efficiency is defined as

$$U(x) = \sum_{k \in \mathcal{K}_1} U_k(x) + w \sum_{k \in \mathcal{K}_2} U_k(x), \tag{1}$$

where $\mathcal{K}_1$ and $\mathcal{K}_2$ are sets containing the class 1 (express passenger) and class 2 (ordinary passenger) trains, respectively, $U_k(x)$ is the utility accrued from train $k$, and $w = 0.4$ is a weight that controls the priority given to class 1 trains in comparison to class 2 trains. The priority given to class 1 trains by the value of $w$ reflects the fact that class 1 trains typically carry more passengers. Furthermore, class 1 trains usually complete longer journeys and hence delays to class 1 trains can have a greater impact in terms of reactionary delay outside the geographical scope of the TTRP instance.

The utility $U_k(x)$ accrued from train $k$ is calculated as a weighted average across the set $J^k$ of timetabled events for train $k$ within the area and time horizon modelled:

$$U_k(x) = \sum_{j \in J^k} \beta_k^j U_k^j(x). \tag{2}$$

Each event $j \in J^k$ in the timetable for train $k$ corresponds to a particular part of the track, and a time that train $k$ is due to enter it. These can include arrival events at platforms and passing events at junctions or key points along a route. The values of $\beta_k^j$ ensure that more important events, such as an arrival into a major station or exiting the modelled area, are weighted more highly than events at minor stations. When alternative platforms are available, these are separate events $j \in J^k$. For events $j$ at platforms other than the originally scheduled platform, $\beta_k^j$ is 0.9 times the weight at the originally scheduled platform. This discourages platform changes.

The efficiency $E(\mathbf{x})$ of a set of solutions $\mathbf{x} = (x^i : i \in I)$ to the whole set of instances $I$ can be calculated by summing the individual efficiency of each solution. Denoting the efficiency function $U$ when applied to each instance $i$ as $U_i$, this can be written as

$$E(\mathbf{x}) = \sum_{i \in I} U_i(x^i). \tag{3}$$

## 2.2 Fairness

For a given instance, let the set of operators be $O$, and let $\mathcal{K}_o \subset \mathcal{K}$ be the set of trains that are operated by operator $o$. The efficiency function $U(x)$ can be rewritten as

$$U(x) = \sum_{o \in O} U_o(x), \tag{4}$$

where

$$U_o(x) = \sum_{k \in \mathcal{K}_1 \cap \mathcal{K}_o} U_k(x) + w \sum_{k \in \mathcal{K}_2 \cap \mathcal{K}_o} U_k(x) \tag{5}$$

is the part of the efficiency arising from trains operated by $o$. Since $U_o(x)$ includes only trains from operator $o$, it can be used to measure the utility of operator $o$.

The utilities $U_o(x)$ are difficult to compare because there might be different numbers of trains with different weights in the instance. For each operator $o \in O$, let $x_o^*$ be an optimal solution when the objective is to maximise $U_o(x)$, rather the total utility $U(x)$ over all operators. Each solution $x_o^*$ represents the best solution operator $o$ can hope for, and $U_o(x_o^*)$ provides an upper bound for $U_o(x)$. This allows us to calculate a normalised utility for each operator

$$\hat{U}_o(x) = \frac{U_o(x)}{U_o(x_o^*)}. \tag{6}$$

These values can be compared between operators. A value of $\hat{U}_o(x) = 1$ indicates that operator $o$ realises their maximum possible utility in the rescheduled solution $x$ — all events for all trains are due to be carried out on time and as planned. Conversely, $\hat{U}_o(x) < 1$ indicates that one or more events have been cancelled or rescheduled to occur on a different platform, or late.

A social welfare function (such as $\alpha$-fairness, Atkinson (1970)) could be applied to the set of utilities $\{\hat{U}_o(x) : o \in O\}$ to measure the fairness of the solution $x$ for a single instance. This would allow fairness to be formulated as an objective function so that fairness-maximising solutions to individual instances could be computed to solve the TTRP. It would also open the possibility of using multi-criteria methods to balance the objectives of maximising fairness and maximising efficiency within each instance.

However, when considering operator fairness for timetable rescheduling, it is problematic to focus on single hour-long instances separately. This is because operators experience fairness and unfairness over a much longer period of time. The operation of a TTRP algorithm on a railway is likely to involve solving hundreds of different instances, involving repeated allocations of track capacity between the same sets of operators. Instead, it is much more appropriate to consider many consecutive instances of the problem as a single, combined allocation problem. That is the approach taken in this paper.

Considering fairness over a whole instance set rather than on an individual instance basis has important implications for fairness. It means that each individual instance need not be fair, provided any operators that loose out can be compensated in other instances. This is crucial when one considers that a typical TTRP instance considers changes to the timetable over a time horizon of only one hour. Many instances involve only a small number of decisions such as which train should go ahead of the other out of a pair of conflicting trains. It may be impossible to resolve such problems in a fair way, or doing so may require a large degradation in efficiency. Our approach of considering fairness over the whole instance set overcomes this issue.

Consider a set of solutions $\mathbf{x} = (x^i : i \in I)$ to a set of instances $i \in I$. We index the previous notation by $i$ so that $O_i, U_{i,o}, \hat{U}_{i,o}$ and $x_o^{i,*}$ correspond to the notation $O, U_o, \hat{U}_o$ and $x_o^*$, respectively, when applied to each instance $i$. Note that the set of operators $O_i$ can be different across instances.

The normalised aggregated utility for operator $o$ over $I$ can be calculated as

$$\hat{U}_o(\mathbf{x}) = \frac{\sum_{i \in I : o \in O_i} U_{o,i}(x^i)}{\sum_{i \in I : o \in O_i} U_{o,i}(x_o^{i,*})}. \tag{7}$$

These values are then used in the $\alpha$-fairness welfare function to produce our measure of fairness:

$$F_\alpha(\mathbf{x}) = \begin{cases} \sum_{o \in O} \dfrac{\hat{U}_o(\mathbf{x})^{1-\alpha}}{1-\alpha} & \alpha \geq 0, \alpha \neq 1 \\ \sum_{o \in O} \log \hat{U}_o(\mathbf{x}) & \alpha = 1. \end{cases} \tag{8}$$

# 3 RESULTS

An area of railway around Doncaster station has been used as a case study for evaluating fairness. Definitions of any railway signalling terminology used below can be found in (Reynolds *et al.*, 2020). Doncaster station lies on the East Coast Main Line, a busy railway corridor connecting London with Leeds, York, Newcastle and Edinburgh. The wider area covered also contains portions of four double track lines that all begin at Doncaster and go towards Sheffield, Lincoln, Leeds and Hull, respectively. The area lies within a single area of signalling control, and contains 225 berths with 313 valid berth transitions. The station itself has 9 platforms and 85 track circuits. Doncaster station is an important interchange for a variety of inter-city and local services operated by seven different operators. It is also a busy bottleneck, with over 30 trains per hour at peak times. This makes it ideal for investigating the interactions between different operators.

The data for the case study comes from January 2017. Seven different passenger operators run services through the area during this month. Operators NT and LNER operate 41% respectively 35% of the 10,027 trains in the period. More than 80 % of NT's trains are class 2 and these trains make up 93% of all class 2 trains. All trains operated by LNER and four other operators are class 1.

The month of January 2017 is split into 310 non-overlapping hour-long instances of the TTRP (between 8am and 6pm each day, for 31 days). These instances are created from real historical data about the timetable, and the traffic perturbations that actually occurred. The number of operators running trains in each instance ranges from two to six, with the most common number being five. By using instances that cover a whole month, we are able to understand fairness over the whole month, rather than on an instance-by-instance basis.

For each of the 310 instances, an efficiency-maximising solution was calculated using a solving time limit of 600 seconds. Only 12 instances were not solved to optimality within this time limit. For these instances, the best solution found during the time limit was selected, which was less than 1% away from optimality in all cases.

The normed aggregated utility $\hat{U}_o(\mathbf{x})$ ranged between 0.996 and 0.999. However, the proportion of instances, where $\hat{U}_{o,i}(x_i) = 1$ was between 41.3 % and 88,9 %. In particular the value of 41.5% for one operator was considerably lower considerably lower than for all other operators, so that there is inequality in these figures. We also observed that the $\alpha$-fairness of the set of instances is better than that of some individual instances, but on the whole fairer instances are more numerous than less fair ones.

To understand sources of unfairness we further analysed the pairwise trade-offs between operators. We furthermore studied the influence of parameter $w = 0.4$ (see Section 2.1) on the computation of fairness. To this end, we re-solved all instances twicw, namely with values $w = 0.7$ and $w = 1$ and observed that increasing $w$ increases fairness and decreases efficiency. To conclude, we observed that there is unfairness in solutions to the TTRP problem,and that the value of $w$ has a considerable influence on the level of unfairness.

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