

# Energy-optimal Public Transport

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## 1 INTRODUCTION

There are several possible objective functions for public transport planning. While passenger convenience and operational costs are the most common objectives in the well researched sequential solution process, other objectives such as the robustness of the obtained public transport system are considered in the literature as well, see e.g. Friedrich *et al.* (2017).

In this work we investigate the objective to minimize the energy needed to serve the public transport system while still considering the passenger convenience. Note that we are not only considering the special case of electric vehicles, see e.g. Perumal *et al.* (2021) but want to find a general public transport system that consumes a lower amount of energy than a system obtained by the algorithms in literature. This may as well be a system operated by traditional fuel-driven buses where we hence want to find a solution consuming less fuel.

To introduce energy in the public transport optimization process, we first need to define the elements used in public transport planning. We assume that a public transport network (PTN) is given, consisting of stops  $V$  and direct connections  $E$  between them. Lines are then defined as simple paths on the PTN that should be served regularly in a planning period. We assume that a set of possible lines, the line pool  $\mathcal{L}$ , is given as well. Then *line planning* is the problem of finding a set of lines with frequencies such that all given passengers can be transported. For an overview on line planning, we refer to Schöbel (2012). For a given line plan, we can assign (periodic) arrival and departure times for each line at its respective stations. This process is called timetabling, for an overview see Lusby *et al.* (2011). Afterwards, the periodic timetable can be rolled out to cover a given planning period (e.g. extend a periodic timetable for one hour to a day), where each line serving defines a trip that needs to be covered by a vehicle. *Vehicle scheduling* describes the process of finding an assignment of vehicles to trips, where each vehicle serves a list of trips, with potential empty trips connecting the end of one line to the start of the next line. We refer to Bunte & Kliewer (2009) for more details and an overview on different models to solve this problem.

## 2 DIFFERENT ABSTRACTIONS FOR ENERGY-BASED OPTIMIZATION

There are different problem stages and levels of detail to consider when we want to allow the optimization process to respect the energy needed to operate the created public transport system. We discuss possible models based on the data available. For this, we adapt models for line planning and vehicle scheduling. Since the model for computing the energy consumption of a vehicle in detail is computationally difficult (and non-linear), see e.g. [Speckert \*et al.\* \(2014\)](#), we use abstractions that allow us to approximate the energy needed using linear integer optimization models.

### 2.1 Planning based on altitude data

Information that is easy to obtain for a given infrastructure network is altitude data for the edges  $e \in E$ . To approximate the energy needed to traverse an edge  $e = (v, w)$ , we use the upward altitude difference, i.e., the positive altitude difference that needs to be traversed when traveling from  $v$  to  $w$  along  $e$ . We call this  $u(e)$ . As a baseline cost function we choose the cost function  $\text{cost}(l)$  for each line  $l \in \mathcal{L}$  from [Gattermann \*et al.\* \(2017\)](#), i.e.,

$$\text{cost}(l) = c_l \sum_{e \in l} \text{len}(e) + c_e |\{e \in l\}| + c_f, \quad (1)$$

where  $c_l$ ,  $c_e$  and  $c_f$  are given weight factors and  $\text{len}(e)$  is the (given) length of an edge  $e \in E$ . We now extend this formulation by adding the upward altitude difference as well:

$$\text{cost}_a(l) = c_a \sum_{e \in l} u(e) + c_l \sum_{e \in l} \text{len}(e) + c_e |\{e \in l\}| + c_f. \quad (2)$$

This now allows us to use known line planning algorithms such as cost-based models or weighted sum methods containing the line costs to optimize w.r.t. the upward altitude difference used by the lines, resulting in less energy needed for the computed public transport system. Additionally, we may adapt the weighting factors  $c_a$ ,  $c_e$ ,  $c_f$  and  $c_l$  to change the focus between operational costs and energy consumption.

We can extend this approach to the vehicle scheduling stage as well. Again, the baseline cost function that we use to compute the costs of a vehicle schedule is a weighted sum, i.e.,

$$\text{cost}(t) = c_d \text{dur}(t) + c_s \text{len}(t), \quad (3)$$

where  $c_d$  and  $c_s$  are weight factors,  $\text{dur}(t)$  is the duration of the trip  $t$  (based on the current timetable) and  $\text{len}(t)$  is the length of  $t$ . Note that the length can be computed based on the length  $\text{len}(e)$ ,  $e \in E$  of all edges contained in the trip, i.e.,  $e \in t$ . This is based on the line (for service trips) or on the edges used in the empty trip. Again, we can extend this formulation to

$$\text{cost}_a(t) = c_a \sum_{e \in t} u(e) + c_d \text{dur}(t) + c_s \text{len}(t). \quad (4)$$

Note that for the overall costs of a vehicle schedule, we consider the number of vehicles needed as well but this term is not dependent on a single trip and therefore not present in (4).

With this, we are now able to use models from the literature to do energy-aware cost optimization in both line planning and vehicle scheduling. For a computational evaluation of this modelling propositions, see Section 3.

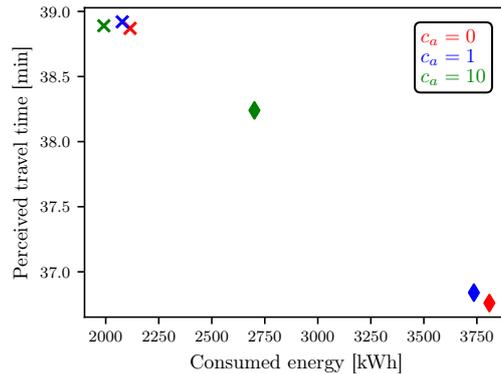


Figure 1 – Comparing different solutions for their energy-efficiency using the approach of Section 2.1 on the bus system in Göttingen. Solutions marked with a cross are computed using a cost-oriented line planning algorithm and solutions marked with a diamond are computed using a weighted sum approach of costs and direct travellers. Direct travellers are passengers that can travel from their origin to their destination without transferring between lines.

## 2.2 Planning based on simulation data

Instead of using the altitude approximation described in Section 2.1 we can use more detailed information as well. For this, we assume that we have a black-box tool to simulate the energy needed for a given path in the infrastructure network. This could e.g. be the simulation module of the tool “Geo-referenced Analysis and Virtual Measurement Campaign (VMC)”, see [Speckert et al. \(2014\)](#), which is used for this work. Again, we discuss possibilities to include such simulation results in line planning as well as in vehicle scheduling.

For line planning, using the simulation allows us to approximate the energy needed to serve every single line very accurately. We call this  $\text{en}(l)$ . We can therefore replace the altitude term in (2) and obtain

$$\text{cost}_{\text{en}}(l) = c_{\text{en}}\text{en}(l) + c_l \sum_{e \in l} l(e) + c_e |\{e \in l\}| + c_f. \quad (5)$$

For vehicle scheduling, the exact energy consumption of service trips is important to determine the overall energy consumption but since all service trips need to be covered and are fixed, we do not need these values for the optimization process. Instead, we use the simulation to approximate the energy used for empty trips in more detail. We need to find energy-efficient shortest paths between all end and start stations of lines. To do so, we use a linear objective function, namely

$$w(e) = w_a u(e) + w_l l(e), \quad (6)$$

to approximate the energy needed for each edge  $e \in E$  of the PTN using the altitude data from Section 2.1. Since (6) is linear and non-negative, we can now use known shortest paths algorithms from literature to obtain all possible empty trips  $t \in \mathcal{T}$ . Afterwards, every  $t \in \mathcal{T}$  can be simulated to obtain  $\text{en}(t)$  and we adapt (4):

$$\text{cost}_{\text{en}}(t) = c_{\text{en}}\text{en}(t) + c_d \text{dur}(t) + c_s l(t). \quad (7)$$

## 3 FIRST COMPUTATIONAL RESULTS

We used the open-source software framework LinTim, see [Schiewe et al. \(2021, n.d.\)](#), to test our adaptations in algorithms from the literature and the bus system in Göttingen, and VMC, see [Speckert et al. \(2014\)](#), for simulating the energy consumption of lines and vehicle schedules.

Using the adaptations proposed in Section 2.1, we obtain the results presented in Figure 1. For this, we preprocessed the infrastructure in VMC to obtain altitude data. Note that this can be done very efficiently since no simulation is needed. Afterwards, we used a cost-oriented and a weighted-sum (cost and direct travelers) integer programming approach to compute two line plans for three different weight scenarios. Then a periodic timetable, a vehicle schedule and the cost function (4) were computed. We can observe that for the cost-oriented approach all three weight factors perform very well with regard to their consumed energy but still the solutions with higher altitude weight are able to outperform solutions with lower altitude weight, with a maximal effect of around 6%. But this effect is even stronger for the weighted-sum line plans, allowing to reduce the energy needed to perform the resulting public transport system by around 30%. Of course, this energy saving is not for free, the corresponding perceived passenger travel time is increased by around 4%. Note that the perceived passenger travel time is a weighted sum of the travel time and the number of transfers needed and the passengers are routed using a shortest path approach w.r.t. their perceived travel time.

Similar effects can be seen when using the adaptations from Section 2.2, since the energy consumption can be approximated even better. But this also increases the computation times, since we need to compute more simulations using VMC, which is time consuming.

## 4 CONCLUSION

In this work, we presented different approaches to incorporating energy consumption into public transport planning and showed their effectiveness with first computational results. An important next step is to incorporate the timetabling process as well. Here, we adapted the stages line planning and vehicle scheduling but the timetable plays an important role in the energy consumption of a public transport system as well, since the energy consumption depends on the speed of driving. Another aspect would be a further coupling to the simulation software, i.e., allowing the optimization direct access to the simulation itself or important parts of it instead of only using the output to improve the linear objective functions.

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