Mode share equilibrium with tradable credit scheme over different time cycles

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1 INTRODUCTION

Most cities around the world are coping with congestion. It induces economic losses, contributes to global warming, and increases the risks of respiratory diseases. Several demand management schemes are investigated in the literature to reduce the number of individual cars on the network. The Tradable Credit Schemes (TCS) require users to have credits to access a given resource, here traveling by private car. The goal is to foster Public Transportation (PT) usage. Those credits are issued by the regulating entity and can be freely traded with other users. In opposition to pricing, the credit price is not set by the regulator but determined by the market. See Lessan & Fu (2019) for an overview of TCS.

Most of the TCS in the literature assumes the credits have to be used on the day they are emitted and cannot be stocked. However, a few works investigated TCS in a framework allowing the credits consumption over several days. In Ye & Yang (2013), the credits are allocated for several days, and the price is updated each day based on the number of credits still available. Tian & Chiu (2015) defines consumption periods for the TCS and the users need to balance their credit account by the end of the period by using the credit market. If they fail, they need to fill the gap by buying credits at a high price from the regulator. The framework of Miralinaghi & Peeta (2016) allows the users to report credits between periods, subject to a fee. Miralinaghi & Peeta (2019) specifies the multi-period TCS to foster the shift from conventional cars to low-emission ones. In Miralinaghi & Peeta (2020) the authors account for the perception of the future prices by the network users.

In most of those works, the congestion model is based on the Bureau of Public Roads (BPR) function. As the BPR function is a static model, the travel time depends only on the number of cars using the link and does not account for congestion dynamics. As an alternative, we use here the trip-based Macroscopic Fundamental Diagram (MFD) concept (Mariotte *et al.* (2017), Lamotte & Geroliminis (2018)), which considers the congestion dynamics at a large scale and takes into account the heterogeneity of the trip lengths.

This paper investigates the equilibrium of a trip-based MFD under a TCS over several days with different validity periods. The demand is elastic as we account for modal choice: car or PT. The need for driving private cars may vary over days, e.g., when picking up someone or buying groceries. To account for this, riding PT on given days induces a penalty. The users can spend their credits over a validity cycle of several days. The novelty lies in the simultaneous consideration of the congestion dynamics with an MFD framework, the report of credits, and the heterogeneity of the days.

2 METHODOLOGY

The users are aggregated into N groups. Each group i consists of γ_i travelers, has a fixed trip length l_i and departure time t_i. Its degree-of-freedom is the ratio of car users per day $x_{d,i}$. As we are using a trip-based MFD framework, the travel time per car T_i of the group i is defined by:

$$l_i = \int_{t_i}^{t_i+T_i} V(t)\mathrm{d}t, \tag{1}$$

where $V(t)$ is the mean speed on the network at time t. We assume the travel time per PT $T_{i,\mathrm{PT}}$ is independent of the accumulation and depends only on the Origin-Destination (OD) pair. The costs for each group for a day d is given by:

$$\begin{cases} C_{i,\mathrm{car}}(d) &= \alpha T_i(\mathbf{x_d}) + (\tau - \kappa)p; \\ C_{i,\mathrm{PT}}(d) &= \alpha T_{i,\mathrm{PT}} + \mu_i(d) - \kappa p, \end{cases} \tag{2}$$

where α is the value-of-time (VoT), τ the credit charge: the number of credits needed to drive a car, p the credit price, κ the allocation: the number of credits each traveler gets for free from the regulator, and $\mu_i(d)$ the penalty for using PT on day d. It represents a day-specific crucial need to use the car. The decision process is based on logit. The ratio of group i which *wants* to take the car is:

$$\psi_{d,i}(\mathbf{x_d}, p) = \frac{e^{-\theta C_{i,\mathrm{car}}(d)}}{e^{-\theta C_{i,\mathrm{car}}(d)} + e^{-\theta C_{i,\mathrm{PT}}(d)}}, \tag{3}$$

with θ the coefficient of the logit.

This work focus on the modal shares at equilibrium over a cycle of c days. The users can spend their credits on any day during a given cycle. Then looking at the equilibrium for one day is not enough. We need to consider credit consumption during the cycle to enforce the credit cap: the total number of consumed credits cannot exceed the number of allocated credits. The leftover credits are lost once the cycle is over. We compute the simultaneous equilibrium of several days forming a cycle. The credit price is defined by the market-clearing condition (MCC): the price is zero or all the issued credits are consumed. The equilibrium over a cycle of c days is formalized with the following set of equations:

$$\begin{cases} \psi_{\mathbf{d}} & = \mathbf{x_d} \ \forall \ d \in [1, c]; \\ p\left(\sum_{i=1}^{N}\sum_{d=1}^{c}\gamma_i(\kappa - \tau x_{d,i})\right) & = 0; \\ \sum_{i=1}^{N}\sum_{d=1}^{c}\gamma_i(\tau x_{d,i} - \kappa) & \leq 0. \end{cases} \tag{4}$$

Note that choosing $c = 1$ is equivalent to forbidding credit carry-over as the credits are then valid only for a day.

A cost function is formulated to minimize the gap between the modal shares and logit-based decisions. At the same time, the MCC should hold. It is added into the cost function to avoid non-affine constraints for computational purposes. The quadratic cost function is defined as

$$J = \frac{1}{2}\sum_{d=1}^{c}\sum_{i=1}^{N}(x_{d,i} - \psi_{d,i})^2 + \eta\frac{1}{\sum_{i=1}^{N}\gamma_i}p\left(\sum_{i=1}^{N}\sum_{d=1}^{c}\gamma_i(\kappa - \tau x_{d,i})\right), \tag{5}$$

with η the coefficient related to the MCC.

The computation of the quadratic function is based on the linearization of the travel times with respect to the modal shares. One major contribution of this work is quantifying the delay induced by one user to the users (a.k.a. marginal external cost) in a trip-based MFD framework. See (Balzer & Leclercq (2022)) for more details.

3 PRELIMINARY RESULTS

The proposed methodology is applied to a case study based on the city of Lyon. The network of Lyon is represented by one reservoir (Mariotte *et al.* (2020)). The scenario is based on the 7:00-8:00 demand. It represents 115 628 users aggregated into 1314 groups with 224 different OD pairs. The PT travel times are retrieved from the navigator HERE. We consider a horizon of ten days. A random draw is generated for each group, and each day to attribute days with a non-zero penalty. The distribution (common for all groups) is given in Fig. 1a. The equilibriums over the horizon are computed for different cycle lengths: one, two, five, and ten days for the whole horizon. It means the horizon consists of several cycles depending on its length. For example, with a cycle length of two days, the horizon of ten days consists of five cycles. The credit charge τ is set to 200 credits and the allocation κ to 100 credits. It means the regulator authorizes at most half of the users to drive their cars on average over the cycle. It is equivalent to what is expected with a license plate rationing scheme.

We define the social cost of the travel as the monetary equivalent of the total travel time plus the penalty. The number of car users, credit prices, and social costs are represented in Fig. 1. The TCS reduces the social cost (travel time plus penalty) by about 10%. The credit price is

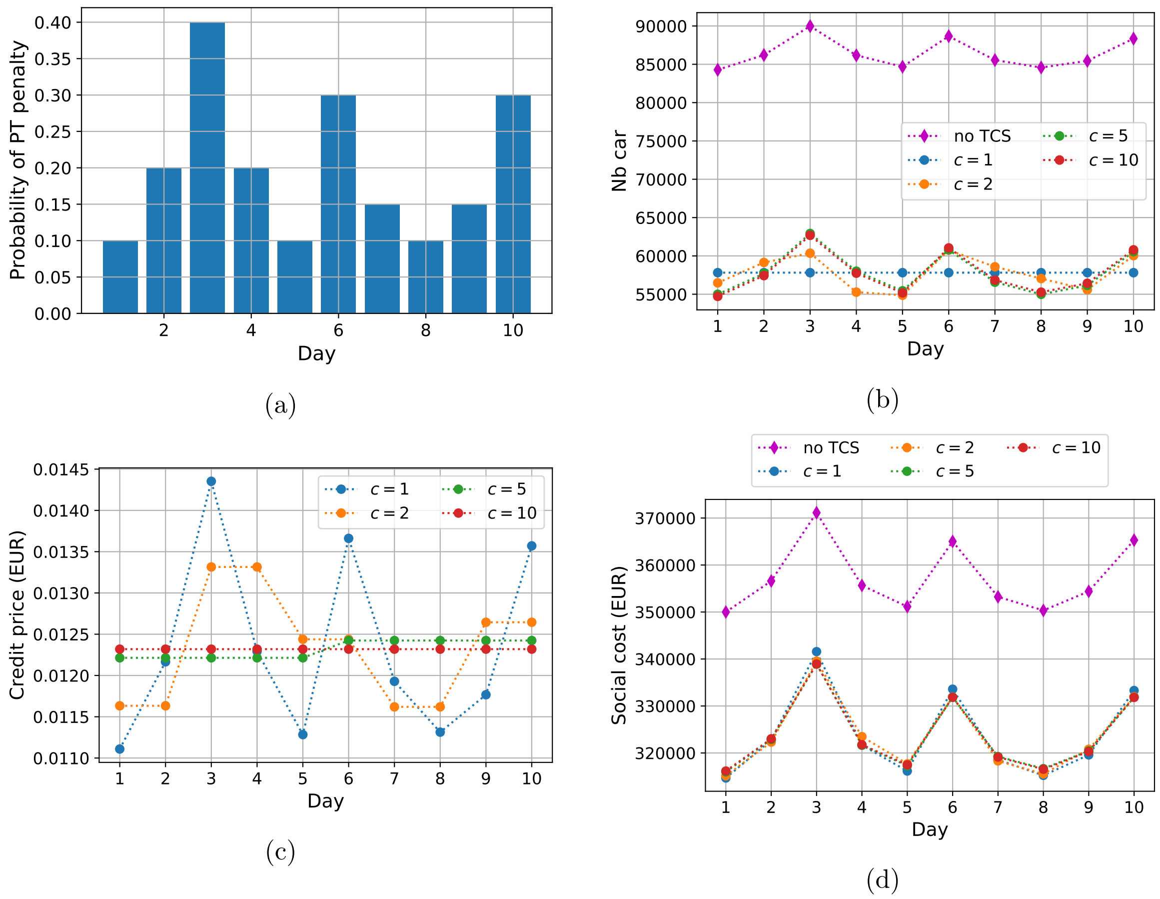

Figure 1 – *Equilibrium for an horizon of 10 days with different cycle lengths: (a) probability of PT penalty, (b) number of car users, (c) credit price, and (e) social cost.*

acceptable as the toll equivalent $p(\tau - \kappa)$ is around 1.2 EUR. For a cycle of one day, the price is highly correlated with the penalty for taking PT: the credit price increases by about 30% between day one and day three. As expected, it is high when many users need to drive their cars. Indeed, the demand is high, and the offer is limited as the issued credits are valid only for the current day. The TCS is more flexible as the cycle length increases. The number of car users varies. The credit price is more stable across the days and less volatile. However, the number of cars driving is more variable, but it does not significantly modify the social cost.

4 DISCUSSION

We set up a TCS framework with trip-based MFD to study different credit consumption cycle lengths. On some days, some users suffer from a penalty if they take the PT. We present a methodology to compute the modal equilibrium of a TCS permitting cycle-based credit consumption. We assess the effect of credit cycles on credit price and social costs with a numerical example based on the morning commute in Lyon. Allowing credit carry-over does not significantly affect the performance of the TCS. It can be an asset when it goes to the acceptability of the TCS: the credit price is more stable over days, and the credit consumption is flexible. Moreover, it makes it possible to use its car for a day without spending a single euro, only by saving credits from the other days. Future investigations include the comparison of TCS against other demand management policies.

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