A column generation heuristic for the Pickup and Delivery Problem with Online Transfers

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1 Introduction

Recently there is increasing attention in the literature for more flexible and autonomous ways of public transportation. Some focus on semi-flexible public transportation. Examples are systems with a mix of conventional taxis and timetabled shuttle busses (Horn, 2002), limiting the number of transfers in a transit system with a large number of vehicles (Cortés & Jayakrishnan, 2002), and integrating fixed routes with a general pickup and delivery problem (Aldaihani & Dessouky, 2003).

In this research, we focus on full flexible passenger transportation. This can be considered as a variant or extension of the Dial-a-Ride Problem (DARP). The focus of the DARP is to bring passengers from their origin to their destination (e.g. the classical taxi problem). In later variants of this problem shared door-to-door services were investigated, to see the effect of allowing for multiple passengers (with different origins and destinations) to share the ride. We want to go one step further and look at the effects of allowing for transfers of passengers in such a shared door-to-door service, both *online* transfers that take place within two driving vehicles that are platooning, as well as by using *outside* transfers.

The use of transfers in passenger transportation is not new. In public transport, it is one of the aspects that is typically considered when making a line plan (Schöbel, 2012). In logistics delivery services, transferring goods between vehicles at cross-docks or hubs is already quite common. In courier (pickup and delivery) type of services (Shang & Cuff, 1996, Sampaio *et al.*, 2020) we see transfers of passengers. However, these approaches mostly consider transfers at fixed predefined locations. In some of these approaches vehicles need to be synchronized, i.e. at the same location at the same time, to transfer the goods from one delivery vehicle to another, and in other approach this might not be required as temporary storage is allowed so a vehicle can leave goods for another vehicle to pickup later.

In order to investigate the potential of *online* transfers (i.e. while moving) in shared door-to-door services, many modelling tricks commonly used in the Dial-a-Ride Problem break down. Typically the only information about the road network that is required are distances between pickup and delivery locations, and in extensions of locations where transfers are possible. In case of online transfers, the actual topology of the road network is required, as facilitating an online transfer between two vehicles requires that both vehicles share the same road segment at the same point in time, even though the previous and next pickup and delivery locations are completely distinct. While many models for the Dial-a-Ride Problem focus on the sequence of the pickups and deliveries of the customers, online transfers require an integrated approach that takes the exact routes, timing included, of both passengers and vehicles into account.

In our research, we consider a Mixed Integer Program (MIP) to solve this problem, that can either be solved directly for small instances, or provide the basis for a heuristic in medium sized instances. We also develop a related column generation approach, and perform extensive computational experiments. Finally, we provide a number of analytical insights into the possible advantage of allowing online transfers, but these are omitted from this extended abstract for reasons of brevity.

As far as we know, only Fu & Chow (2022) investigated a closely related problem, the Pickup and Delivery Problem with Synchronized En-Route Transfers. An efficient heuristic is proposed, as well as a MIP model. However, as opposed to our model this model works with a single type of transfers, which have to be performed on nodes, ensuring enough time is scheduled to allow a transfer, while our model supports both online transfers that can take on arcs and outside transfers that take place on nodes (but require that the vehicles stop). Furthermore, our model is different since we employ a time-expanded network, rather than Tucker-Miller-Zemlin style subtour elimination constraints.

2 Methodology

We use a MIP formulation based on a time-expanded network, where a considered time window is discretized into time units. We assume that demand has strict time windows in which the demand must either be served or rejected at the cost of a penalty. The formulation can be regarded as a multi-commodity flow where flows associated with the demand consume capacity, whereas the flow associated with the vehicles provides capacity. To enforce that passengers can transfer in practice, and are not able to transfer when two vehicles cross paths on a node without forming a platoon, we work with consecutive arc constraints to force that vehicles are platooning while online transfers take place. Similarly, we use consecutive arc constraints to enforce that vehicles stop at a node to ensure that a passenger can enter or exit the vehicle. These approaches do assume that these actions can be performed in one time unit, which requires that the time discretization works with large enough time units.

The column generation approach we developed works by decomposing paths of vehicles and passengers out of the MIP model. The columns can be generated by dynamic programming, which boils down a shortest path algorithm where you need to track the previous arc to deal with the duals of the consecutive arc constraints. Unfortunately, the column generation seems to suffer from symmetry and/or degeneracy. With stabilization techniques and employing it as a heuristic with a time limit, we are still able to outperform the direct formulation in a number of medium sized instances, but for smaller instances the direct formulation seems the preferred choice for many instances.

3 Experiments and Results

We perform extensive computational experiments. Since many common datasets for DARP only provide a distance matrix, but do not provide the topology of the road network, we used an implementation of a model proposed by Courtat *et al.* (2011) to generate ten road networks of 8, 10, 12, 15 and 20 road segments. We generate a set of smaller instances with the following pairs of numbers of passengers and vehicles: $(5,2),(10,2),(15,3),(20,3)$ or $(40,6)$. The vehicles have 4 seats each. Furthermore, we have a set of larger instances with either 50 or 60 passengers which have to be served by either 8, 10, 12 or 14 vehicles. We use these instances to compare the objective value of solving the MIP model with a time limit against solving the heuristic. We observe that for the smaller instances, sometimes the MIP approach is better and sometimes the column generation works better, but for the larger instances, the column generation approach performs better in almost all cases. This comparison is visualized in Figure 1.

We then look at instances where many online transfers are used, to see how the service is designed by our method. Figure 2 contains a visualization of one example solution that makes extensive use of online transfers. In Figure 2b it is interesting to observe that there seem to be two main phases in which the bulk of the passengers are transported. For the majority of passengers, they are travelling either in the first half of the second half of the figure. However, four passengers that make an online transfer from the blue to the yellow vehicle seem to link these phases. It is our aim to investigate the solution in more detail. Additionally, we also provide some analytical insights that tell us in which cases online transfers can be beneficial.

4 Conclusion

We introduce a MIP model and a column generation model for the Pickup-and-Delivery Problem with Online Transfers. We have performed a large number of computational experiments with small and medium sized instances, where we indeed see online transfers occuring. We intend to investigate the benefits and potential of online transfers in more depth using the developed methodology.

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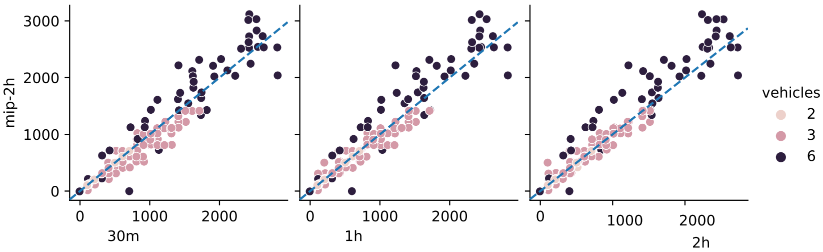
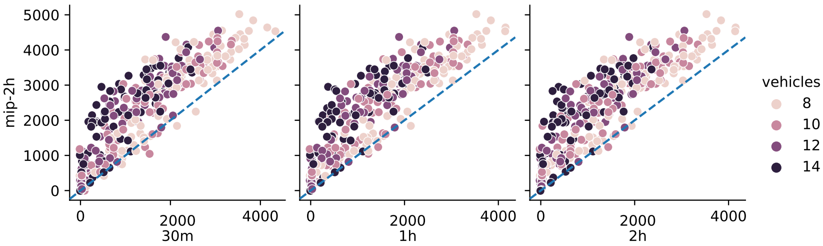
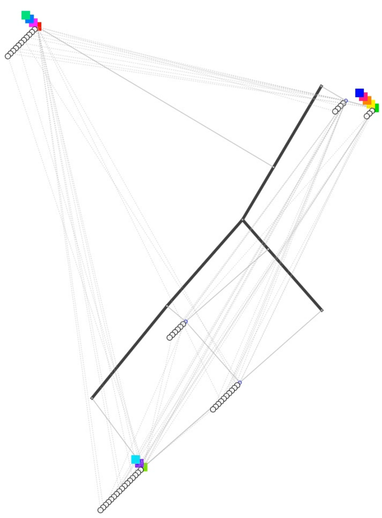


(a) *Comparison for the smaller instances.*

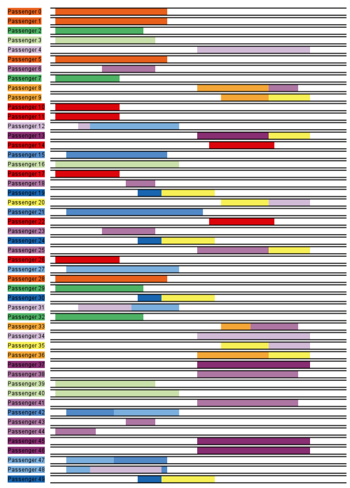


(b) *Comparison for the larger instances.*

Figure 1 – *Comparison of a column generation heuristic against solving a MIP model directly. The MIP has a time limit of two hours, the column generation procedure is stopped either after thirty, sixty or hundred twenty minutes. Each point represents an instance that was solved, and the (minimized) objective for different methods is reported. Points above the diagonal indicate the column generation heuristic finds better solutions than the MIP.*



(a) *Visualization of the network*



(b) *Visualization of vehicle occupancy by passengers*

Figure 2 – *Example instance and a found solution that services all passengers by utilizing a large number of online transfers. In Figure 2a, the thin dashed lines connect the origin and destination of circle-shaped passengers. The thick lines indicate road segments where stopping is not possible, the thin lines are road segments where stopping at nodes is possible. In Figure 2b each row indicates a passenger and the x-axis indicates time. The colored parts indicate at which intervals passengers are travelling inside a vehicle, the color indicating the exact vehicle. Passengers served directly have only a single color in their row, whereas two different colors indicate one online transfer, and three different colors indicates two online transfers. A movie of this solution can be accessed via* https://doi.org/10.5281/zenodo.5855693

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