Freight on Transport problem: Extended formulation and column generation approach

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1 INTRODUCTION

The last-mile delivery is the most expensive part of the whole freight delivery process, in addition to being the most unsustainable one. The delivery system affects not only the shipping companies, but urban life as well. There are several stakeholders in the last-mile delivery system, either direct or indirect, who are impacted. The impact caused by vehicles performing deliveries associated with last-mile operations can be three-fold: economic, social and environmental (Viu-Roig & Alvarez-Palau, 2020). To keep up with the growing demands and resulting issues caused by e-commerce delivery systems, companies are looking towards innovative approaches that reduces the costs of social and environmental externalities. In this work we consider the strategy based on the use of Public Transport Services for delivery operations. The effectiveness of an integrated delivery system on public transit has already been demonstrated in several cities (see Marinov *et al.* (2013), Saito (2021), Cleophas *et al.* (2019)). Amazon is also looking towards using public buses for its deliveries, and has received a patent that would transform buses into parcel carriers (Baron, 2019, Reul, 2019).

In this paper, we study a problem which we call as the Operational Freight on Public Transport (OpFOT hereafter) problem. It consists of a three-tier package delivery system. In the first tier (also referred as T1), the packages are delivered from the Consolidation and Distribution Center (CDC) to nearby stops of the public transportation vehicles from which they can be picked up by these vehicles. We call these stops *drop-in* stops. The second tier (T2) of the delivery is the one that occurs on-board public vehicles, which have pre-determined schedules, itineraries and stops. The vehicles pick the packages from the drop-in stops and transport them to some other stops on their routes, which we call *drop-out* stops. The stops where the packages are dropped by the vehicles are within the city and are close to the customer locations. Finally, the city freighters pick the packages from the drop-out stops and deliver them to their respective customers, using sustainable and green modes of transport, like electric vehicles, drones, bikes, or even freighters simply walking for the delivery. This constitutes the third and final tier of the problem (T3). The goal is to determine the routes in T1 and T3 such that the corresponding cost is minimized. To the best of our knowledge, we are the first to study this three-layer Freight-on-Transport delivery system.

2 Problem definition

The OpFOT problem was introduced in Mandal & Archetti (2021) and is formally described as follows.

A set $\mathcal{C}$ of customers is defined where each customer $c \in \mathcal{C}$ requests the delivery of one parcel of size $q_c \geq 0$ during a delivery time window $[T_c^e, T_c^l]$ at a location of her/his choice. We assume that all delivery requests are known in advance and that all parcels start their journey at a Consolidation and Distribution Centre (CDC), denoted with o.

In the first tier, parcels are moved from the CDC to public transport stops in the set $\mathcal{S}_{in}$ through a set $\mathcal{D}$ of homogeneous trucks of capacity $\mathcal{Q}_d$. Each time a truck moves from a location i to a location j, it incurs a cost c_{ij} which is proportional to the distance between i and j.

In the second tier, each parcel travels between two bus stops, aboard a bus. Specifically, parcels are picked up from a bus stop in $\mathcal{S}_{in}$ and delivered to a stop in $\mathcal{S}_{out}$. We denote the bus fleet as $\mathcal{P}$. Each bus $p \in \mathcal{P}$ has capacity Q_p and serves a route $\mathcal{S}_p = (s_p^1, \ldots, s_p^{|\mathcal{S}_p|})$ represented by the ordered list of stops the bus visits.

In the third tier, freighters move parcels between bus stops and customer locations. We denote with $\mathcal{K}$ the set of homogeneous freighters with capacity $\mathcal{Q}_f$ who can perform the deliveries to the customers. Each freighter $k \in \mathcal{K}$ is assigned to a stop $s_k \in \mathcal{S}_{out}$ where she/he can start and end his service. Freighter travel time between locations $i, j \in \mathcal{S}_{out} \cup \mathcal{C}$ is denoted as l_{ij}. Each freighter route has a maximum duration of $L_{\max}$, which includes both the travel time and service times T_c incurred when delivering parcels to customers $c \in \mathcal{C}$. In addition, as for trucks in Tier 1, freighters incur a cost c_{ij} when moving from a location i to a location j which is proportional to the distance, but might be different than the cost associated with trucks. Indeed, as freighters typically use light vehicles to perform deliveries (e.g., bikes) the associated traveling cost is typically much smaller than the one related with trucks.

The OpFOT problem asks to design the routes of both delivery trucks and freighters and determine which bus should carry each package, in order to deliver every package according to capacities and time constraints, while minimising the cost of the designed routes. Note that no cost is associated with Tier 2, i.e., with the use of public transport. This is because buses will be used anyway for passenger transportation.

3 An extended IP formulation

Let $\mathcal{R}_d$ (resp. $\mathcal{R}_f$) be the set of feasible routes for delivery trucks from the depot to stops (resp. for freighters from a stop to the customers). Each route $r \in \mathcal{R}_d \cup \mathcal{R}_f$, is associated with the set of packages transported by the vehicle, the cost of the route c_r as well as the departure time of the route. We introduce a binary variable $x_r \in \{0,1\}$ for each truck route $r \in \mathcal{R}_d$ which states whether route r is part of the solution or not. Analogously, we use a binary variable $y_r \in \{0,1\}$ for each freighter route $r \in \mathcal{R}_f$. To properly model the transport of the parcels by the public buses, we employ binary variables $z_{pcs}^{in} \in \{0,1\}$ to state whether bus $p \in \mathcal{P}$ picks up the parcel of customer $c \in \mathcal{C}$ at stop $s \in \mathcal{S}_{in}$, and variables $z_{pcs}^{out} \in \{0,1\}$ which states whether bus $p \in \mathcal{P}$ drops the parcel of customer $c \in \mathcal{C}$ at stop $s \in \mathcal{S}_{out}$. In the following, we will define as $\mathcal{S}^c$ as the stops which can be used to serve customer $c \in \mathcal{C}$ and $\mathcal{S}_p$ as stops served by bus $p \in \mathcal{P}$. Our formulation for OpFOT consists in the minimisation of the costs of the selected routes subject to several constraints ensuring that: (i) the selected routes on the 3 echelons are compatible, (ii) the resources (e.g., capacities, etc.) on each echelon are not exceeded, and (iii) every customer receives its parcel on time.

Objective function:

$$\min \sum_{r \in \mathcal{R}_d} c_r x_r + \sum_{\mathcal{R}_f} c_r y_r. \tag{1}$$

Constraints:

- Maximum number of trucks:
$$\sum_{r\in\mathcal{R}_d} x_r \leq |\mathcal{D}|. \tag{2}$$

- Maximum number of freighters:
$$\sum_{r\in\mathcal{R}_{f_s}} y_r^s \leq |\mathcal{K}|. \tag{3}$$

- Each parcel must be moved by one bus:
$$\sum_{p\in\mathcal{P}_c}\sum_{s_1\in\mathcal{S}_p\cap\mathcal{S}^c_{in}} z^{in}_{pcs_1} = \sum_{p\in\mathcal{P}_c}\sum_{s_2\in\mathcal{S}_p\cap\mathcal{S}^c_{out}} z^{out}_{pcs_2} = 1 \quad \forall c \in C. \tag{4}$$

- A bus either do both pick up and drop off a parcel or none of these:
$$\sum_{s_1\in\mathcal{S}_p\cap\mathcal{S}^c_{in}} z^{in}_{pcs_1} = \sum_{s_2\in\mathcal{S}_p\cap\mathcal{S}^c_{\text{out}}} z^{out}_{pcs_2} \quad \forall c \in C,\ \forall p \in \mathcal{P}_c. \tag{5}$$

- Bus capacities are respected:
$$\sum_{c\in C} q_c \sum_{j\leq i}\left(z^{in}_{pcs^j_p} - z^{out}_{pcs^j_p}\right) \leq Q_p \quad \forall p\in\mathcal{P},\ \forall i\in\{1,\ldots,|\mathcal{S}_p|-1\}. \tag{6}$$

- Some route must have dropped a parcel which is picked up at an in-station:
$$z^{in}_{pcs_1} \leq \sum_{r\in\mathcal{R}_{d_{pcs_1}}} x_r \quad \forall c\in C,\ \forall p\in\mathcal{P}_c,\ \forall s_1\in\mathcal{S}^c_{in}\cap\mathcal{S}_p. \tag{7}$$

- Some route must be delivering a parcel which is dropped off at an out-station:
$$z^{out}_{pcs_2} \leq \sum_{r\in\mathcal{R}^f_{pcs}} y_r \quad \forall c\in C,\ \forall p\in\mathcal{P}_c,\ \forall s_2\in\mathcal{S}^c_{out}\cap\mathcal{S}_p \tag{8}$$

  where $\mathcal{R}^f_{pcs}$ are all the freighter routes routes delivering package c from stop s, starting at a time compatible with route r.

We note that the methodology proposed in the following can be adapted to different problem setting, like different cost functions for vehicles used in the first and third layer. We also note that capacity constraints of trucks and freighters are handled in the corresponding pricing subproblems.

3.1 The pricing subproblems

As the above formulation may have exponentially-many variables, we propose to tackle the solution of its linear relaxation with a column generation approach. To this end, both sets of x- and y-variables shall be dynamically generated during the solution process according to the values in the current optimal solution of the restricted master problem. The pricing subproblems associated with x and y variables can be stated as elementary shortest path problems with resource constraints (ESPPRC) in which the cost of visiting a node depends on different aspects which we briefly describe in the following.

Pricing variables x. In the case of x variables, i.e., truck routes from the depot dropping packages at the stops, the cost of visiting a stop s depends both on the packages which are dropped at s and the time at which the stop is visited. We propose two approaches to solve this problem: *(i)* build a graph in which nodes represent pairs $(s, c) \in \mathcal{S} \times \mathcal{C}$ and solve a *time-dependent* ESPPRC, and *(ii)* build a graph in which nodes represent triples $(s, c, p) \in \mathcal{S} \times \mathcal{C} \times \mathcal{P}$ and solve an ESPPRC problem *with time windows*.

Pricing variables y. In the case of y variables, i.e., freighter routes from a stop dropping packages at final destinations, the pricing can be solved independently on each potential stop. From a stop $s \in \mathcal{S}$, the cost of visiting a customer c depends on the time at which the route starts (not on the time the customer is visited). However, the cost function is piece-wise constant. Based on this fact we partition the time horizon at the points in which this function changes, and solve one subproblem for each of these time segments restricting the starting time of the route to the corresponding time interval. Each subproblem is solved as a standard ESPPRC problem *with time windows*.

4 Solution approach

Once the column generation presented above terminates, i.e., it does not find any new negative reduced cost column, then the optimal solution of the linear relaxation of the extended formulation is found. We then use the complete set of columns generated to determine a feasible solution to the OpFOT problem as follows. We build an instance of the extended formulation with the set of columns found and solve the corresponding Mixed-Integer Linear Program (MILP). This procedure is known as *restricted master heuristic* and widely used in column generation approaches.

5 Computational tests

We tested the approach presented above on the instances proposed in Mandal & Archetti (2021) and compare it with the problem formulation as well as the three decomposition approaches presented in this paper. Preliminary tests show that the approach proposed in this paper is competitive. However, results highlighted that the pricing problem associated with Tier 1 is extremely cumbersome. Thus, the research is now focused on accelerating this phase and, eventually, finding more effective strategies for the pricing. Also, tests will be made to assess whether a compact formulation of Tier 1 would be more efficient.

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