On the fundamental diagram of signal controlled urban roads

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1 INTRODUCTION

Fundamental diagrams, as their name indicates, are fundamental analysis tools in the field of transportation. Transportation researchers have attempted to elucidate these functions for urban areas for the last five decades. This paper addresses this challenge and builds our understanding of how urban congestion forms and propagates.

Fundamental diagrams (FDs) are link (i.e., segment, roadway) models that describe at an aggregate level the relationship, for a given road, between two out of the three fundamental traffic variables: (space-mean) speed, flow and density. The FD has been, and continues to be, the most important traffic modeling tool since the emergence of traffic data collection and flow modeling. The seminal work of Greenshields (1935) is, arguably, considered as the work that founded and pioneered the field of traffic flow theory (Kühne, 2008). The work proposed the first FD that stipulated a linear relationship between speed and density. The majority of the FD analysis has focused on uninterrupted traffic, including highways. The assumption of uninterrupted traffic is violated in most urban roadways that are controlled by traffic lights or signs (e.g., stop, yield signs). Research on FD formulations for interrupted traffic of urban signalized roads is limited (Elmar Brockfeld & Wagner, 2008, Wagner *et al.*, 2009, Wu *et al.*, 2011, Dakic & Stevanovic, 2018).

Past work on signalized urban roadways has attempted to map density to speed (Wu *et al.*, 2011). Given the difficulty and cost of collecting density data, commonly used proxies include occupancy (Wu *et al.*, 2011) and degree of saturation (Pascale *et al.*, 2015), estimated from stop-bar (or stop line) detectors. For these detectors, the queue-over-detector (QOD) phenomenon can make the proxies unreliable estimates of density. Hence, the work of Wu *et al.* (2011) proposed a mechanism to remove the impact of QOD. The method involves the use of both advance detectors (placed well upstream, e.g., 400 feet, of the stop-line), stop-bar detectors, and detailed signal event data. Advance detectors can yield more reliable occupancy estimates (Hall *et al.*, 1992). The work of Dakic & Stevanovic (2018) recognized the broad availability of stop-bar detectors and proposed the use of density saturation as a more reliable density proxy. The estimation of density saturation requires both stop-bar detector data along with detailed signal plan event data.

Our proposed work overcomes these challenges by proposing a mapping between the more broadly available flow data to speed. The flow data is robust to the detector placement, as it is not impacted by the QOD phenomenon. The commonly available stop-bar detectors can be used to reliably estimate flow. Moreover, it is a direct estimation of the traffic state variable, as opposed to resorting to the use of a proxy.

Additionally, unlike past work, this work shows that an FD can be derived without knowledge of demand on competing movements nor of detailed signal event data (e.g., phase actuation times, phase durations). This is unexpected, since the flow of an actuated movement relies on demand for other lower and higher priority movements at that intersection, as well as on the specific actuation logic of an intersection (Wu *et al.*, 2011).

To the best of our knowledge, past FD insights for interrupted urban traffic have considered a two-to-one mapping between flows and densities, or equivalently between flows and speeds. In other words, a given flow measurement is consistent with two traffic states (i.e., two different speed values or two different density values). We postulate that these mappings are one-to-one. Moreover, the functional form we use is closely related to one commonly used for highways. Together these findings contradict past understandings and findings in the field. For instance, Elmar Brockfeld & Wagner (2008) state: "We hypothesize that in fact there is a fundamental diagram in urban roads, however, there is no one-to-one correspondence to the fundamental diagram on freeways." Our current results indicate plenty of similarities with freeway or highway FDs.

2 METHODOLOGY

2.1 Proposed fundamental diagram (FD)

To formulate the FD, we consider the major street of an actuated signalized intersection. We introduce the following variables: (space-mean) speed v, flow q; and the following parameters: maximum speed (i.e., speed limit) v_{max}, flow capacity q_{cap}, FD power coefficients α and β. The proposed FD has the following functional form:

$$v = v_{max} \left(1 - \left(\frac{q}{q_{cap}}\right)^{\alpha}\right)^{\beta}. \tag{1}$$

To facilitate comparison across roadways with different speed limits, we rewrite Eq. (1) as:

$$\frac{v}{v_{max}} = \left(1 - \left(\frac{q}{q_{cap}}\right)^{\alpha}\right)^{\beta}. \tag{2}$$

Eq. (1) is inspired from the commonly used functional form for highway FDs that relate speed and density (rather than flow) (May & Keller, 1967):

$$v = v_{min} + (v_{max} - v_{min}) \left(1 - \left(\frac{k}{k_{jam}}\right)^{\alpha}\right)^{\beta}, \tag{3}$$

where k represents the density, v_{min} the minimum speed and k_{jam} jam density.

For highways, the FD mapping between speeds and density is a one-to-one mapping (e.g., Eq. (3)). This leads to a one-to-two mapping between speeds and flow. Our findings on urban signal controlled roads indicate that the speeds to flow mapping is one-to-one (Eq. (1)). This implies that the measurement of flow, which is the most commonly available type of measurement, is sufficient to infer the traffic state. This is an important finding since flow data is the most commonly available traffic data in urban areas.

2.2 Data sources

We consider all movements of the major street of an urban traffic actuated intersection. The definition of major street is obtained from the Utah Department of Transportation (UDOT)

Automated Traffic Signal Performance Metrics (ATSPM) (Utah Department of Transportation (UDOT), 2020). No signal plan data, such as phase durations or actuation information, is required to derive the FD. Through the ATSPM portal, we have access to lane-level vehicular count (i.e., flow) data obtained at stop-bar detectors, aggregated at the roadway-level (i.e., aggregated across lanes) and at the hourly level. We use aggregated and anonymized space-mean speeds of a given roadway by looking at Google Maps driving trends (Lau, 2020). The speed estimates are obtained for the entire roadway, they are not lane-specific. Space-mean (i.e., harmonic mean) speeds are obtained for each volume interval. The volume values are segmented into intervals of 30 veh/hr. To enable comparison across segments and intersections, we work with a normalized speed that considers the ratio of speed to speed-limit, as defined by Eq.(2). The speed limit of each roadway is obtained from Google Maps.

3 RESULTS

Figure 1 considers an actuated intersection. Each of the row of plots considers one of the two travel directions of the major street. For each plot, the x-axis displays the flow (in veh/hour-lane) and the y-axis displays the (unitless) normalized speed as defined in Eq.(2). In the first column of plots, the average measurements curve is in blue with error bars that have a total width of two standard deviations. The fitted FD, based on Eq.(2), is in yellow. The blue curve indicates a flow-to-speed mapping that is one-to-one. Fitting of the yellow curve does not depend on or require any signal data.

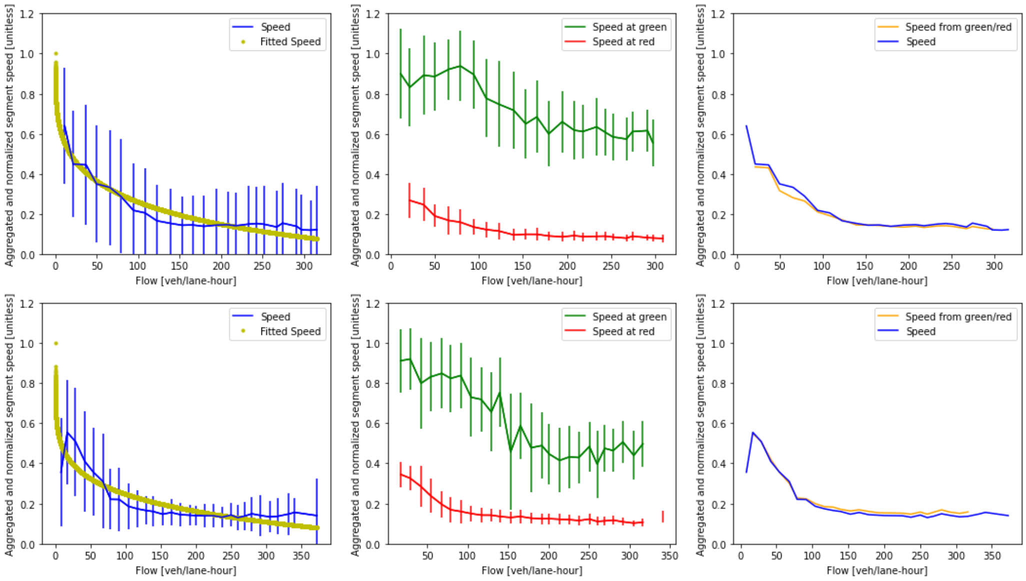

Figure 1 – *Fundamental diagrams of the two travel directions of the major street of an actuated urban intersection.*

The second column of plots consider FDs that exploit signal plan information. The green (resp. red) curves consider only speed measurements observed while the thru-movement phase has a green (resp. red) signal. These second column plots illustrate how the FD of Figure 1 that is derived without any signal data (i.e., first column plots) can be decomposed into two signal-specific FDs. These green and red FDs can also be approximated by the functional form of Eq.(2). The corresponding fitted curves are not displayed for the sake of clarity. For a given row of plots, the error bars of the green and red FDs tend to be smaller than those of the blue FD. This illustrates how the large error-bars of the blue FD are mainly due to the large speed differences between the red signal phase (where vehicles are mostly stopped) and the green signal phase (where signals are mostly moving).

In the third column of plots, the blue curve is the same curve as that of the first column of plots. The yellow curve is obtained from a weighted combination of the data that is used to define the green and the red FDs of the second column. The third column of plots illustrates how for a given travel direction (i.e., a given row of plots), the blue FDs of the first column plots can be seen as a mixture of the green and the red FDs of the second column. The two weight parameters of this mixture are estimated as the proportion of observations that fall in the green signal phase, and the proportion that fall in the red signal phase.

More detailed analysis of our findings, as well as the relationship between highway FDs and signal-controlled urban FDs will be presented and discussed at the conference.

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