Dynamic schedulling auction using time-expanded decision diagram

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1 Introduction

Recent advances in information technology have led to the development of new transportation services. In a tradable permit system as a method to improve the operational efficiency of these services, it is assumed that permits are distributed to users, mutual transactions are conducted among users, and users purchase permits to use transportation services. Several tradable permit system models have been proposed in extant studies (Yang and Wang; 2012, Wu, et al.; 2012, Akamatsu and Wada; 2017, Hara and Hato; 2017). In general, there are many issues to be addressed in terms of extensibility to system dynamics and traffic assignment models.

In a tradable permit system, the operational efficiency is increased as a result of mutual trading of permits. However, the allocation is dependent on transactions and autonomous optimal matching is almost impossible. We introduce the auction mechanism as a method for determining the allocation of permits. The auction system is a mechanism for autonomously achieving a desirable allocation of goods in society.

2 The Framework and the Solution Method

Assuming that traffic flow in the near future is sufficiently predictable and the road network can be time-structured by using timesteps to represent time discretely. The number of vehicles that can pass through each edge in each timestep can be calculated. In this study, the permit is corresponds to each edge in the time-structured network as in Fig.1.

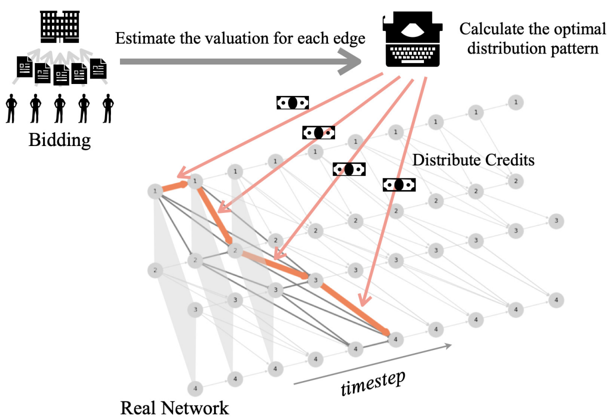

Figure 1 – *Distribution of permits to a capacity-constrained time-structured network*

When allocating permits based on an auction, the service provider needs each user's value function for permits. However, it is almost impossible for users to directly express their valuations to a lot of edges in a time-structured network. We devised a system for mechanically estimating the value function based on a Discounted Recursive Logit(dRL) model. The immediate utility and the value function to the destination are processed by the Bellman equation. The service provider interpret the utility for each edge as a valuation from each user. With accumulated user data, service providers can estimate the value function for the entire time-structured network from limited user information with high accuracy.

An auction is a mechanism that uniquely determines the allocation of goods based on bidding information. The paths in a time-structured network is determined by a scheduling problem. In the system envisioned in this research, bidding information changes sequentially in the form of new bids or schedule changes, so it is necessary to determine the allocation dynamically. This dynamic scheduling auction is based on updated bidding information, and the allocations at each timestep must be consistent with the results of previous toll allocations.

We consider an auction at timestep t. Assuming that all users' valuations for each edge in the time-structured network $\mathcal{G}$ are accurate. We summarize them as a value vector $\boldsymbol{v}_t = \{v_{i,e}\}$. The distribution of the permits is similarly denoted as the allocation vector $\boldsymbol{x}_t = \{x_{i,e}\}$. Let E be the number of all edges in $\mathcal{G}$, V be the number of all nodes, and N_t be the number of all users in the auction at timestep t. Then, optimal allocation problem of permits can formulated as Eqs.(1), (2), (3), (4) and (5); 0-1 integer programming problem.

$$\underset{x_t \in \{0,\ 1\}}{\text{maximize}} \quad \boldsymbol{v}_t \boldsymbol{x}_t \tag{1}$$

$$\text{subject to} \quad \begin{bmatrix} B_1 & & O \\ & \ddots & \\ O & & B_1 \end{bmatrix} \boldsymbol{x}_t = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} B_2 & & O \\ & \ddots & \\ O & & B_2 \end{bmatrix} \boldsymbol{x}_t \leq \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} I_E & \cdots & I_E \end{bmatrix} \boldsymbol{x}_t \leq \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_E \end{bmatrix} \tag{4}$$

$$C_t \boldsymbol{x}_t = \boldsymbol{g}_t \tag{5}$$

Eq.(2) guarantees the continuity of the spatio-temporal transition and Eq.(3) ensures that single or less movement is achieved for each user. Eq.(4) ensures that the allocation pattern satisfies the capacity constraint and Eq.(5) guarantees the consistency with the results of the auction conducted at timestep $t-1$. Here, $B_1 \in Z^{(V-2)\times E}$ is a submatrix of the incidence matrix $A(\mathcal{G})$, which consists of rows other than those corresponding to the origin and destination, $B_2 \in N^{1\times E}$ is the submatrix of $A(\mathcal{G})$ consisting of a row corresponding to the origin, and I_E is the unit matrix of size E. μ_e is the maximum number that can be allocated for edge e in $\mathcal{G}$. Also, C_t is a diagonal matrix, and its diagonal component $c_{i,e}$ and the elements $g_{i,e}$ of the vector $\boldsymbol{g_t}$ are defined by Eq.(6). Let $E'(t)$ be the edge set whose source nodes correspond to a timestep before t, let $E_i^*(t-1)$ be the edge set whose permits were allocated to user i at timestep $t-1$, and let $E_{i,origin}$ be the edge set corresponding to the stay at the origin.

$$c_{i,e} = \begin{cases} 1 & e \in E'(t) \wedge e \notin E_{i,origin} \\ 0 & otherwise \end{cases} \qquad g_{i,e} = \begin{cases} 1 & e \in E'(t) \wedge e \in E_i^*(t-1) \wedge e \notin E_{i,origin} \\ 0 & otherwise \end{cases} \tag{6}$$

Two important problems arise in the process the service provider solves the optimization problem through dynamic scheduling auction. The first problem is the strategy-proofness of the users' bidding. If there are users who intentionally state an invalid valuation through strategic bidding, it is impossible to maximize the social welfare correctly. Therefore, auctions need to be designed so that true bidding is the dominant strategy for each user. The second problem is the large numerical cost involved in solving the optimal allocation problem. This 0-1 integer programming problem is classified as NP-hard. Furthermore, the dynamic scheduling auctions envisioned in this study involve repeated recalculations within a short span of time.

3 VCG Mechanism of Dynamic Scheduling Auction

The VCG mechanism is an auction mechanism designed to motivate agents(system provider and users) to select a socially efficient allocation of public goods. We designed the VCG mechanism to accommodate dynamic scheduling auctions. Here, we show the VCG mechanism for dynamic scheduling auctions achieves efficient allocation and strategy-proofness.

If the user correctly represents his/her true information, the service provider assumed to accurately get the value function through the dRL model, $\boldsymbol{b}_1 \cdots \boldsymbol{b}_t = \boldsymbol{v}_1 \cdots \boldsymbol{v}_t$. First, when the inner product of the correct bidding vector and the allocation vector is maximized with the constraint equations, the state of the system corresponds to the social optimum.

Next, we show the mechanism satisfies the strategy-proofness. Let $\boldsymbol{b}_t$ be the bidding vector at timestep t. The allocation in the auction at timestep t depends on the bidding vector at timestep t, $\boldsymbol{b}_t$, and the bidding vectors for all previous auctions. Therefore, the allocation pattern is denoted as $S(\boldsymbol{b}_1, \cdots, \boldsymbol{b}_t)$. Also, let $W(\cdot)$ denote the social welfare achieved by the auction. Under the VCG mechanism, the payment of user i who placed a new bid at timestep t, $P_i(\boldsymbol{b}_1, \cdots, \boldsymbol{b}_t)$, is expressed as $P_i(\boldsymbol{b}_1, \cdots, \boldsymbol{b}_t) = W(\boldsymbol{b}_1, \cdots, \boldsymbol{0}, \boldsymbol{b}_{t',-i}, \cdots, \boldsymbol{0}, \boldsymbol{b}_{t,-i}) - W_{-i}(\cdot)$ where $\boldsymbol{b}_{t,-i}$ denotes the bidding vector except user i. $W_{-i}(\cdot)$ indicates $W(\cdot)$ minus user i's value $\boldsymbol{b}_{t,i} \cdot S_i(\boldsymbol{b}_1, \cdots, \boldsymbol{b}_t)$.

Suppose a situation where all auction participants except user i bid true value vector $\boldsymbol{b}_{-i} = \boldsymbol{v}_{-i}$. Consider a domination strategy for user i. The surplus of user i is as follows:

$$\begin{aligned}
&\boldsymbol{v}_i \cdot S_i(\boldsymbol{v}_1, \cdots, \boldsymbol{b}_{t',i}, \boldsymbol{v}_{t',-i}, \cdots, \boldsymbol{b}_{t,i}, \boldsymbol{v}_{t,-i}) - P_i(\boldsymbol{v}_1, \cdots, \boldsymbol{b}_{t',i}, \boldsymbol{v}_{t',-i}, \cdots, \boldsymbol{b}_{t,i}, \boldsymbol{v}_{t,-i}) \\
&= \boldsymbol{v}_i \cdot S_i(\boldsymbol{v}_1, \cdots, \boldsymbol{b}_{t',i}, \boldsymbol{v}_{t',-i}, \cdots, \boldsymbol{b}_{t,i}, \boldsymbol{v}_{t,-i}) + W_{-i}(\cdot) - W(\boldsymbol{v}_1, \cdots, \boldsymbol{0}, \boldsymbol{v}_{t',-i}, \cdots, \boldsymbol{0}, \boldsymbol{v}_{t,-i}) \\
&= \boldsymbol{v} \cdot S(\boldsymbol{v}_1, \cdots, \boldsymbol{b}_{t',i}, \boldsymbol{v}_{t',-i}, \cdots, \boldsymbol{b}_{t,i}, \boldsymbol{v}_{t,-i}) - W(\boldsymbol{v}_1, \cdots, \boldsymbol{0}, \boldsymbol{v}_{t',-i}, \cdots, \boldsymbol{0}, \boldsymbol{v}_{t,-i})
\end{aligned}$$

$W(\boldsymbol{v}_1, \cdots, \boldsymbol{0}, \boldsymbol{v}_{t',-i}, \cdots, \boldsymbol{0}, \boldsymbol{v}_{t,-i})$ is independent of user i's bid. The dominant strategy for user i is to maximize $\boldsymbol{v} \cdot S(\boldsymbol{v}_1, \cdots, \boldsymbol{b}_{t',i}, \boldsymbol{v}_{t',-i}, \cdots, \boldsymbol{b}_{t,i}, \boldsymbol{v}_{t,-i})$. Given the allocation pattern is calculated with the 0-1 integer programming problem, the following equation holds for $\boldsymbol{b}_{t',i}, \cdots, \boldsymbol{b}_{t,i}$.

$$\boldsymbol{v} \cdot S(\boldsymbol{v}_1, \cdots, \boldsymbol{b}_{t',i}, \boldsymbol{v}_{t',-i}, \cdots, \boldsymbol{b}_{t,i}, \boldsymbol{v}_{t,-i}) \geq \boldsymbol{v} \cdot S(\boldsymbol{v}_1, \cdots, \boldsymbol{v}_t)$$

The dominant strategy for user i is to make an honest bid so that $\boldsymbol{b}_i = \boldsymbol{v}_i$.

4 Calculation Algorithm and Numerical Resluts

The 0-1 integer programming problem consisting of Eqs.(1) - (5) is NP-hard and computationally demanding. Dynamic auctions require frequent recalculations. Although we have succeeded to proof that linear relaxation can be applied to the formulation, the proof is omitted here.

The optimal allocation of permits is a combinatorial optimization problem, and the feasible region is discrete. By enumerating the entire feasible region and indexing them by the social welfare, it is possible to determine the solution. However, because the size of the feasible region grows exponentially due to the combinatorial explosion, the computer cannot handle the memory load when using the general counting method. we use ZDD (Zero-suppressed Binary Decision Diagram) which has high compression performance, especially for sparse combinatorial sets. The advantage of this method is that it can significantly reduce the numerical cost for recalculation. By updating the ZDD sequentially, the number of enumerations can be greatly reduced while maintaining solution continuity. In this study, we follow the flowchart shown in Fig.2 to solve the optimal allocation problems in dynamic scheduling auctions.

We compare two methods, the enumeration-based method with ZDD and linear relaxation. The computation time for the winner-determination problem in the auction at each timestep is shown in Fig.3. The enumeration method using ZDDs shows high computing performance against recalculation while the computation time increases monotonically with the method using linear relaxation as time progresses and the number of participants in the auction increases.

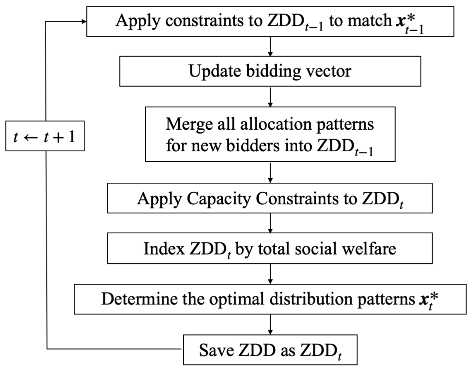

Figure 2 – *Flowchart of the method with ZDD*

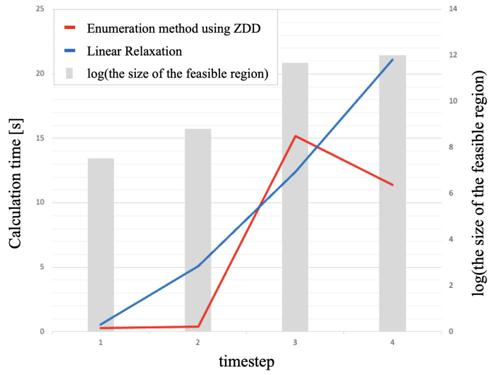

Figure 3 – *Change in calculation time*

5 Conclusion

This study examined tradable permit mechanism with dynamic scheduling auctions as a method for sequential optimization of traffic flow for automated vehicles. There are two problems to achieve the socially optimum. The first is strategy-proofness and the second is the high demand for computational performance. In order to solve the first problem, we proposed a VCG mechanism for dynamic scheduling auctions. This mechanism allowed the system to satisfy the strategy-proof and efficient allocation for all sequential auctions. In order to solve the second problem, we proposed the ZDD-based complete enumeration method. This method shows high computing performance against recalculation while the scale of system is small. Moreover, the successful formulation allows us to obtain the results of large auctions by linear relaxation.

This study considers the optimization of traffic assignment problem with tradable permit system. However, when automated vehicles are introduced, the lanes in the road network will be divided into some types. The optimal allocation problem for lanes will arise. In addition, the computational method proposed in this study is still not scalable to the scale of the system. One of the method is surrogate models, such as the GCN (Graph Convolutional Network) model.

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