Line Planning for Different Demand Periods

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1 MOTIVATION

Line planning and frequency setting is a basic planning step in public transportation for which many models and solution approaches exist (see Schöbel (2012), Kepaptsoglou & Karlaftis (2009)). The goal of line planning is to offer a good service to the passengers while minimizing the costs for the operator. The demand of passengers specifying their travel wishes as origin-destination data is usually given as input.

However, passengers' demand varies within a day: while many passengers want to travel in the morning peak, the demand decreases during the day, and on Sundays or during public holidays demand data is not only smaller than on week-days but also its structure changes significantly. One could accommodate such demand changes by computing a different line concept for each of the given demand periods, but memorizing different line plans with different frequencies within even the same day (lines in the morning peak differ from lines in the afternoon and from lines at night) is not convenient for the passengers. On the other hand, using only one line concept which is able to satisfy the demand during morning peak for the whole day is a waste of resources. Hence, in practice, line concepts differ more or less over the day and the week.

In this talk we develop a model for finding (possibly different) line concept for each of the demand periods, but under the restriction that these line concepts should be *similar*. We propose different definitions for the (dis)similarity of line concepts and add them as constraints, hereby coupling the single line planning problems. We analyze the resulting models and develop solution approaches. In the computational results we demonstrate that maximizing the similarity and minimizing the costs of line concepts are conflicting goals and we show the Pareto front with respect to them.

2 A MODEL FOR LINE PLANNING WITH DIFFERENT DEMAND PERIODS

2.1 Finding cost-optimal line concepts

Line planning has been researched since the paper Patz (1925). There exist different models, see the surveys Kepaptsoglou & Karlaftis (2009), Schöbel (2012), which are still subject of ongoing research. Besides passenger-oriented models which minimize the traveling time of the passengers, cost models are common. Here, a line plan with minimal costs is chosen which has enough capacity to transport all passengers on shortest paths, see Claessens *et al.* (1998) for the original model and Schöbel (2012) for its basic version (Lin). This version is (exemplarily) used as basis of our paper. It is stated next.

Given a public transport network $PTN = (V, E)$ with a set of stations V and direct connections E, the line planning problem (Lin) requires the following input:

- A line pool $\mathcal{L}$ with a set of potential lines, each of them with costs specified by parameters cost_l for all $l \in \mathcal{L}$, and
- lower and upper edge frequency capacity bounds $f_{\min,e} \leq f_{\max,e}$ for all $e \in E$. The lower edge frequency bounds reflect the demand of the passengers, because they ensure that for every edge there are enough lines operated to transport all passengers.

The goal is to choose a set of lines $\mathcal{L}_0 \subseteq \mathcal{L}$ from the pool together with their frequencies $f_l \in \mathbb{N}_0$ such that the frequency bounds are satisfied on every edge and the resulting costs are minimal:

$$
\begin{aligned}
\text{(Lin)}\quad \min \quad & \sum_{l \in \mathcal{L}} \text{cost}_l\, f_l \\
\text{s.t.} \quad & f_{\min,e} \leq \sum_{l \in \mathcal{L}: e \in l} f_l \leq f_{\max,e} \text{ for all } e \in E \\
& f_l \in \mathbb{N}_0 \text{ for all } l \in \mathcal{L}
\end{aligned}
$$

Note that we look for a *line concept*, i.e., a set of lines $\mathcal{L}_0 := \{l \in \mathcal{L} : f_l > 0\}$ together with their frequency vector $(f_l)_{l \in \mathcal{L}} \in \mathbb{N}_0{}^{|\mathcal{L}|}$. A line concept is denoted by $(\mathcal{L}, f)$.

2.2 Finding cost-optimal line concepts for multiple demand periods

We now introduce n different demand periods, each of them given by their specific values $f^{(i)}_{\min,e}, f^{(i)}_{\max,e}$, $i = 1, \ldots, n$ for the lower and upper edge frequency bounds. We allow different line concepts $(\mathcal{L}^{(i)}, f^{(i)}_l)$ for each demand period, but require them to be *similar*. To this end, we propose different functions (see the next section),

$$
\text{dissim}\left((\mathcal{L}^{(i)}, f^{(i)}), (\mathcal{L}^{(j)}, f^{(j)})\right)
$$

telling us how (dis)similar two line concepts $(\mathcal{L}^{(i)}, f^{(i)})$ and $(\mathcal{L}^{(j)}, f^{(j)})$ are. All these measures satisfy that $\text{dissim}\left((\mathcal{L}, f), (\mathcal{L}, f)\right) = 0$ if the two line concepts are identical.

The line planning problem with multiple periods (Multi-Lin) determines not only one frequency f_l for line l but uses variables $f^{(i)}_l$ for all lines $l \in \mathcal{L}$ and all demand periods $i = 1, \ldots, n$:

$$
\begin{aligned}
\text{(Multi-Lin)}\quad \min \quad & \sum_{i=1}^{n} \sum_{l \in \mathcal{L}} \text{cost}_l\, f^{(i)}_l \\
\text{s.t.} \quad & f^{(i)}_{\min,e} \leq \sum_{l \in \mathcal{L}: e \in l} f^{(i)}_l \leq f^{(i)}_{\max,e} \quad \text{for all } e \in E, i = 1, \ldots, n \\
& \text{dissim}\left((\mathcal{L}^{(i)}, f^{(i)}), (\mathcal{L}^{(j)}, f^{(j)})\right) \leq \alpha \quad \text{for all } i, j \in \{1, \ldots, n\} \qquad \textbf{(sim)} \\
& f^{(i)}_l \in \mathbb{N}_0 \quad \text{for all } l \in \mathcal{L}, i = 1, \ldots, n
\end{aligned}
$$

where α specifies how much the line concepts are allowed to differ. A first analysis provides two simple bounds:

- A lower bound is given by ignoring the similarity constraint **(sim)** and solving each of the line planning problems separately for its respective demand period.
- An upper bound is given by using only variables f_l, i.e., the same frequencies for all periods $i = 1, \ldots, n$, hence requiring $f^{(i)}_{\min,e} \leq \sum_{l \in \mathcal{L}: e \in l} f_l \leq f^{(i)}_{\max,e}$ for all $e \in E$ and all $i = 1, \ldots, n$.

3 SIMILARITY OF LINE CONCEPTS

We develop integer programming formulations and analyze their feasibility and complexity for different (dis)similarity concepts. Some of them are described next.

3.1 Frequency-based similarity concepts

In the first class of similarity concepts we require identical lines and only allow differences in their frequencies. We define two line concepts $(\mathcal{L}^{(i)}, f^{(i)})$ and $(\mathcal{L}^{(j)}, f^{(j)})$ to be similar if

- the two sets of lines are identical, i.e., $\mathcal{L}^{(i)} = \mathcal{L}^{(j)}$, and
- the norm of the deviations in frequency is bounded, i.e., $\|f^{(i)} - f^{(j)}\| \leq K$

for some given number K. If $K = 0$ we have $\mathcal{L}^{(i)} = \mathcal{L}^{(j)}$ and $f^{(i)} = f^{(j)}$, i.e., the line concepts are similar if and only if they are identical. With increasing K the similarity decreases.

We use this definition of similarity for specifying **(sim)** in the line planning problem (Multi-Lin). For adding the first requirement, $\mathcal{L}^{(i)} = \mathcal{L}^{(j)}$, we need additional variables x_l specifying whether line l has a positive frequency or not. The second requirement can be included in the linear integer program if we use the sum of absolute deviations $\| \cdot \|_1$ or the maximum absolute deviation $\| \cdot \|_\infty$ as norm. Additionally, we also consider a similarity measure based on the pseudo-norm which counts the number of lines with different frequencies, i.e., the lines in the symmetric difference between the sets $\mathcal{L}^{(i)}$ and $\mathcal{L}^{(j)}$.

3.2 Line-based similarity concepts

For line-based similarity concepts we allow to change the lines themselves between different demand periods. For example, it might be appropriate to add new lines in the morning traffic or not to operate some of the lines on Sundays.

We call a demand structure *monotone* if there exists an ordering of the demand periods such that the lower and upper edge frequency bounds increase for every edge, i.e., if all edges $e \in E$ satisfy that

$$f^{(1)}_{\min,e} \leq f^{(2)}_{\min,e} \leq \cdots \leq f^{(n)}_{\min,e} \text{ and } f^{(1)}_{\max,e} \leq f^{(2)}_{\max,e} \leq \cdots \leq f^{(n)}_{\max,e}.$$

Under this assumption we define two line concepts $(\mathcal{L}^{(i)}, f^{(i)})$ and $(\mathcal{L}^{(j)}, f^{(j)})$ with $i \leq j$ to be similar if they are *nested*, i.e., if

- $\mathcal{L}^{(i)} \subseteq \mathcal{L}^{(j)}$, and
- $f^{(i)} \leq f^{(j)}$.

The similarity constraint **(sim)** then requires that

$$\mathcal{L}^1 \subseteq \mathcal{L}^2 \subseteq \cdots \subseteq \mathcal{L}^n \text{ and } f^{(1)} \leq f^{(2)} \leq \cdots \leq f^{(n)}.$$

For solving (Multi-Lin) we can solve the corresponding integer program, but also an iterative heuristic approach finding the best lines from period to period in a sequential manner has been developed.

As variation of line-based similarity concepts we additionally restrict the number of lines which may differ between two consecutive periods. Another variant of line-based similarity gives special focus to lines that serve parts of a main line. This enforces the usage of more similar lines in contrast to adding completely new lines from the line pool. Based on the cosine similarity (see Li & Han (2013)) we also develop similarity measures for line concepts which take the routes of the single lines into account.

4 EXPERIMENTS

In the experiments we demonstrate that the developed solution approaches work and we show results concerning the trade-off between the costs and the similarity of the line concepts. We use different examples from the LinTim open source library Schiewe *et al.* (2021, n.d.) for conducting the experiments. Figure 1 shows two examples for line concepts with frequency-based similarity where we used the sum of absolute deviations $\|\cdot\|_1$ in the left part of the figure and the maximum absolute deviation$\|\cdot\|_\infty$ on the right.

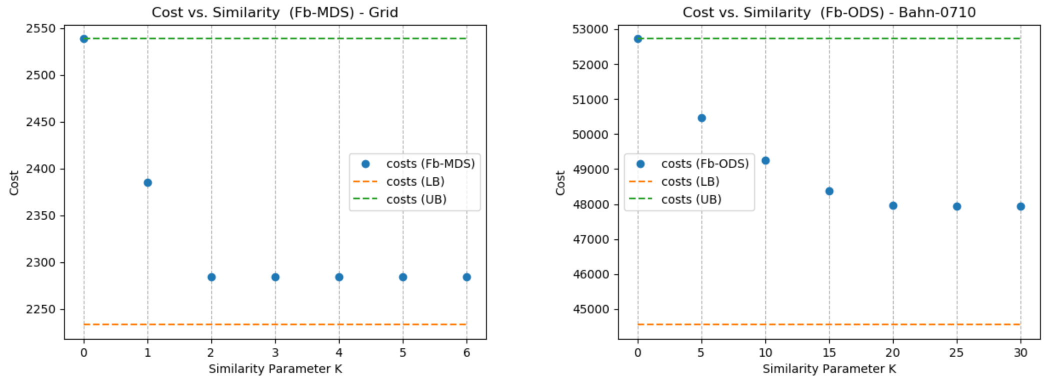

Figure 1 – *Trade-off between costs and similarity for two* LinTim *data sets: Grid and a close-to-real world data set of the German railway system.*

5 CONCLUSION

We conclude that looking at line concepts with respect to different demand periods brings line planning a step closer to real-world applications. It is possible to compute the best possible solution for a given similarity in reasonable computation time for cost-oriented line planning models. A visualization of the trade-off between cost and similarity of the resulting line concepts supports the decision maker to choose a Pareto solution that suits the practical requirements best.

6 REFERENCES

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