A time-dependent analytical traffic model to tackle dynamic Bayesian optimization problems in urban transportation

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1 INTRODUCTION

Most real-world problems in transportation aim to optimize time-dependent decision variables that live in a dynamic, stochastic, and fast-paced environment. Such problems are frequently characterized by a stochastic, nonconvex objective function which has an unknown analytical form. Simulation-based optimization (SO) is a popular method that allows the coupled use of analytical traffic models and complex stochastic urban traffic simulators to address various real-world continuous transportation problems. One approach to tackle SO problems is known as metamodel SO (Osorio & Bierlaire, 2013). In this approach, a limited number of simulation observations is used to fit an analytical approximation of the simulation-based objective function (i.e., a metamodel or surrogate model), which is less expensive to evaluate than the underlying simulator. The fitted surrogate model is employed to perform optimization and derive new local solutions for the decision variables. The performance of new local solutions is successively evaluated by the simulator, leading to new observations that can be used to improve the fit of the surrogate model. Given the computational burden of running simulations, the SO literature has mostly considered stationary surrogate models (i.e. with no time-dependent decision variables and no dynamic information from the simulator). These models can be enhanced through Bayesian optimization (BO, Tay & Osorio, 2021). This leads to the development of efficient dynamic surrogate models that can be integrated into BO to solve dynamic SO problems.

In this work, we formulate a novel analytical and dynamic traffic model. We incorporate the model within a BO framework and use it to tackle a large-scale dynamic SO transportation problem. The proposed traffic model is both realistic and computationally efficient enough so that the resulting BO framework preserves its efficiency and becomes suitable for high-dimensional dynamic problems. This work extends the methodology of Tay & Osorio (2021), which combines a time-independent traffic model with a BO framework, to time-dependent problems through the formulation and use of a time-dependent traffic model. Preliminary results on small-scale scenarios on a toy network from the literature show that incorporating the dynamic analytical traffic model within BO leads to similar or better solutions than using stationary analytical traffic models or no problem-specific information.

2 METHODOLOGY

2.1 Dynamic traffic signal optimization problem

Consider a traffic signal control problem with a fixed cycle time e_i for each link $i \in \mathcal{L}$. The problem optimizes the green splits of each periodic signal phase $j \in \mathcal{P}(i)$ during L time intervals, i.e. $x_{l,j}$. The goal is to determine a fixed time signal plan for each time interval such that the expected travel time of vehicles that end their trips during the second half of the simulation period is minimized (i.e., time interval $l = \lfloor (L/2)+1 \rfloor$ to $l = L$). The problem is modeled as follows:

$$\min_{\mathbf{x}_1,\dots,\mathbf{x}_L} f(\mathbf{x},\mathbf{z};\mathbf{p}) \equiv \frac{1}{\lceil L/2 \rceil} \sum_{l=\lfloor (L/2)+1 \rfloor}^{L} \mathbb{E}[F_l(\mathbf{x},\mathbf{z};\mathbf{p})] \tag{1}$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{P}(i)} x_{l,j} = \frac{c_i - e_i}{c_i}, \ \forall i \in \mathcal{I}, l = 1,\dots,L, \tag{2}$$

$$\mathbf{x}_l \geq \mathbf{x}^{LB}, \quad l = 1,\dots,L, \tag{3}$$

where the objective function (1) is the weighted sum of the simulation-evaluated expected travel time of vehicles during time interval l, i.e. F_l. This depends on the vector $\mathbf{x}_l$ of green splits for time interval l, the vector $\mathbf{z}_l$ of endogenous link-level and network-level metrics for time interval l, and the vector $\mathbf{p}$ of exogenous road network topology and fixed lane attributes. Constraints (2) represent the cycle time constraints and ensure that the sum of all the green splits for a given intersection add up to the proportion of available cycle time c_i that can be optimized (i.e., not fixed). Constraints (3) impose a lower bound on $\mathbf{x}_l$.

Assuming a limited amount of simulation observations, we choose BO as a suitable solution approach (Jones *et al.*, 1998). BO consists of two main components: (i) a model of the objective function, and (ii) an acquisition function. One of the most popular choice of models of the objective function is Gaussian Processes (GP). In this work, prior information is fed to the GP model in the form of a dynamic analytical traffic network model. We refer the reader to Section 3.2 of Tay & Osorio (2021) for a discussion of how this model is incorporated into the GP model.

2.2 Dynamic analytical traffic network model

In this work, each link $i \in \mathcal{L}$ in the network is modeled as a reservoir of vehicles, with inflow $Q^{\text{in}}_{i,l}$ from upstream links and outflow $Q^{\text{out}}_{i,l}$ from to downstream links. The term $\mathbb{E}[F_l(\mathbf{x},\mathbf{z};\mathbf{p})]$ in objective function (1) is approximated by $\sum_{l=1}^{L} \alpha_l f_l^A$, where f_l^A represents the expected time vehicles spend in the network in time interval l. This is approximated using Little's law (Little, 2011), as follows:

$$f_l^A = \frac{\frac{1}{M}\sum_{i\in\mathcal{L}} d_i \sum_{m=0}^{M-1} \rho_{i,l}(m\Delta t)}{\frac{1}{M}\sum_{i\in\mathcal{L}} \sum_{m=0}^{M-1} \gamma_{i,l}\left(1-\frac{\rho_{i,l}(m\Delta t)}{\rho^{\max}}\right)} = \frac{\sum_{i\in\mathcal{L}} d_i \sum_{m=0}^{M-1} \rho_{i,l}(m\Delta t)}{\sum_{i\in\mathcal{L}} \sum_{m=0}^{M-1} \gamma_{i,l}\left(1-\frac{\rho_{i,l}(m\Delta t)}{\rho^{\max}}\right)}, \tag{4}$$

where M represents the number of time steps in interval l, Δt the time step length, $\rho^{\max}$ the maximum vehicle density, and d_i the length of link i. Furthermore, $\rho_{i,l}$ and $\gamma_{i,l}$ represent the vehicle density and the external arrival rate to link i in interval l respectively.

Traffic dynamics are modeled through the following system of dynamical equations:

$$Q_{ij,l}(t) = \frac{\mu_{i,l}(t)}{s} \cdot Q^{\text{FD}}\left(\rho_{i,l}(t)\right) \cdot \mathbf{1}(N^{\text{DQ}}_{i,l}(t) > 0) \cdot p_{ij}\left(1 - \frac{\rho_{j,l}(t)}{\rho^{\max}}\right), \tag{5}$$

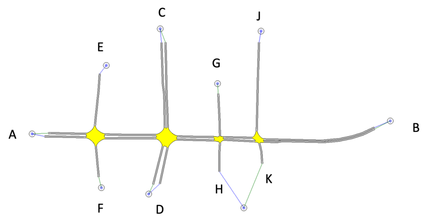


Figure 1 – *Synthetic arterial network model.*

Table 1 – *Demand*

OD Pair	Demand (veh/h)
AB/BA	$1050 \xrightarrow{30\text{ min}} 350$
CD/DC	$300 \xrightarrow{30\text{ min}} 900$
EF	100
HG	100
JK	100

$$Q_{i,l}^{\text{in}}(t) = \gamma_{i,l}\left(1 - \frac{\rho_{j,l}(t)}{\rho^{\max}}\right) + \sum_j Q_{ji,l}(t), \tag{6}$$

$$Q_{i,l}^{\text{out}}(t) = \sum_j Q_{ij,l}(t) + Q^{\text{FD}}\left(\rho_{i,l}(t)\right) \cdot \mathbf{1}(N_{i,l}^{\text{DQ}}(t) > 0) \cdot \left(1 - \sum_j p_{ij}\right), \tag{7}$$

$$N_{i,l}^{\text{DQ}}(t) = N_{i,l}^{\text{DQ}}(t - \Delta t) + \left[Q_{i,l}^{\text{in}}\left(t - t_i^{\text{FF}}\left(1 + \frac{\rho_{i,l}(t)}{\rho^{\max}}\right)\right) - Q_{i,l}^{\text{out}}(t)\right] \cdot \Delta t, \tag{8}$$

$$\rho_{i,l}\left((t + \Delta t\right) = \rho_{i,l}(t) + \frac{1}{d_i}\left(Q_{i,l}^{\text{in}}(t) - Q_{i,l}^{\text{out}}(t)\right)\Delta t. \tag{9}$$

Equation (5) represents the fraction of the saturation flow rate s that can that can flow from link i to link j at a given time $t \in [0, (M-1)\Delta t]$. Here, $Q^{\text{FD}}\left(\rho_{i,l}(t)\right)$ represents the maximum outflow of link i described by the triangular fundamental diagram (Treiber & Kesting, 2013), $\mu_{i,l}$ the service rate, and $\mathbf{1}(N_{i,l}^{\text{DQ}}(t) > 0)$ the indicator function implying that the outflow from link i to link j can only be non-zero at time t if there are vehicles at the downstream end of link i. Note that the outflow depends on the turning probabilities p_{ij}, as defined in Equation (7). The inflows $Q_{i,l}^{\text{in}}$ are set in Equation (6) and the vehicle counter $N_{i,l}^{\text{DQ}}$ is defined in Equation (8), where t_i^{FF} represents the free-flow travel time of link i. Finally, the vehicle density of link i is updated at each time step according to Equation (9). Note that the exogenous variables are s, p_{ij} and $\rho^{\max}$, and $\mu_{i,l}$. The latter depend on the signal plans that are being optimized.

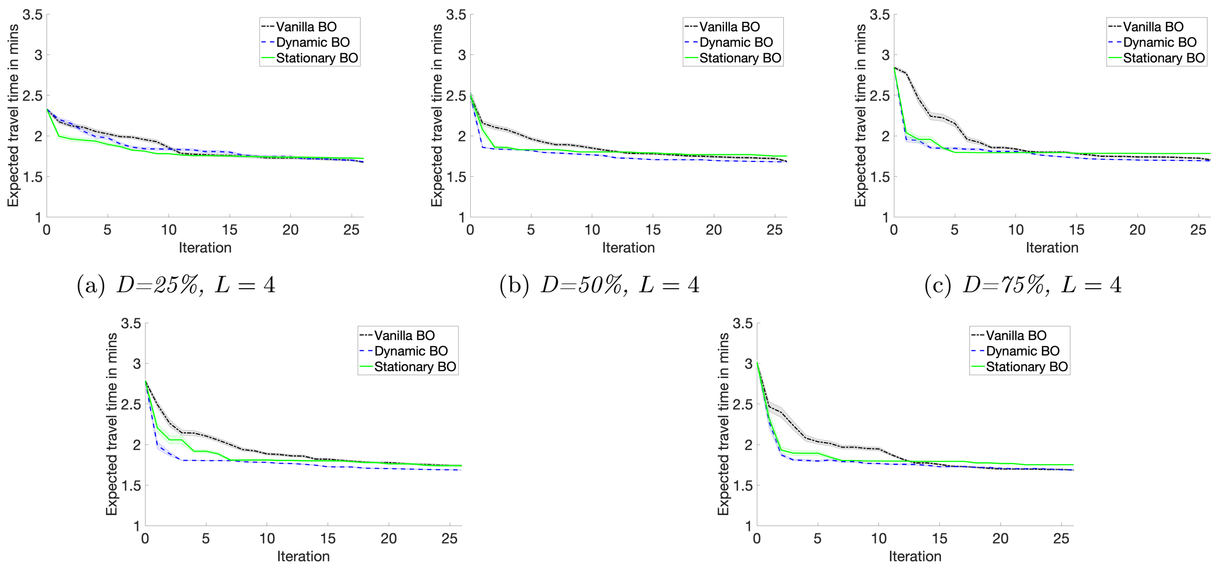
3 NUMERICAL RESULTS

We implement a synthetic toy network consisting of 20 single-lane and 4 intersections, as depicted in Figure 1 (Osorio & Yamani, 2017). The travel demand, shown in Table 1, is defined such that vehicles only have straight paths. We optimize the signal plans for a 1-hour period, with a 15-minute warm-up period. As such, the objective function is the expected travel time of vehicles that end their trips in the last 30 minutes. The minimum green times are set to 4 seconds.

The performance of our Dynamic BO is benchmarked against: (i) Vanilla BO, using no problem-specific prior information; and (ii) Stationary BO, using the stationary traffic model in Tay & Osorio (2021). Scenarios are constructed by varying: (i) the level of demand, as a percentage of the demand in Table 1; and (ii) the number of time intervals. Each BO method is run 3 times for 3 different initial sets, each containing 4 randomly-drawn initial points to fit the GP posteriors. We compute the objective function estimate by taking the mean of 4 simulations, with 2 additional simulations of the best solution at the end of each iteration. Each BO run considers 26 optimization iterations. The relative expected optimization performances of the BO methods are shown in Table 2. Each t-test considers whether Dynamic BO performs better than Vanilla/Stationary BO for a given level of demand and number of time intervals. The null hypothesis assumes that Dynamic BO obtained a mean performance that is worse than or equal to the mean performance of Vanilla/Stationary BO. The t-tests with t-statistics smaller

Table 2 – *One-sided paired t-test results*

Demand	Dynamic BO vs. Vanilla BO				Dynamic BO vs. Stationary BO			
	L = 2		L = 4		L = 2		L = 4	
	t-stat	*p*-value	*t*-stat	*p*-value	*t*-stat	*p*-value	*t*-stat	*p*-value
25%	-1.107	0.150	0.391	0.647	**-1.549**	**0.0800**	**-1.593**	**0.0749**
50%	-0.622	0.276	-0.597	0.283	0.0384	0.515	**-4.078**	**1.77e-3**
75%	**-1.445**	**0.0933**	-0.408	0.347	0.0412	0.516	**-4.082**	**1.76e-3**
100%	0.468	0.674	**-1.695**	**0.0642**	-0.541	0.302	**-2.726**	**0.0130**
125%	**-2.100**	**0.0345**	0.121	0.547	-0.581	0.289	**-4.815**	**6.65e-4**



(a) *D=25%, L* = 4 (b) *D=50%, L* = 4 (c) *D=75%, L* = 4

(d) *D=100%, L* = 4 (e) *D=125%, L* = 4

Figure 2 – *Evolution of the expected objective function value*

than the critical value (-1.397) have their null hypotheses rejected, and are displayed in bold in Table 2. Note that, for $L = 4$, Dynamic BO is always better than Stationary BO on all demand levels. Figure 2 shows the evolution of the best objective function value during the search. Note that Dynamic BO is able to: (i) find good-quality solutions within a few iterations, and (ii) find higher-quality solutions than Vanilla/Stationary BO by the end of the search.

The proposed strategy is currently being tested on a large-scale case study consisting of 698 roads, 2756 lanes, and 444 intersections. We expect to see more benefits to employing Dynamic BO on this more complex network. Additional results obtained by the time of the conference will be included in the presentation.

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